Appendix

3 Optimal Taxation

Suppose there is a revenue maximizing policy maker that chooses a specific tax to be imposed on a monopolist (i.e., $n = 1$). We assume that the policy maker chooses the tax prior to the realization of c_t. The structure of the game is: i) the policy maker chooses a specific tax, τ_t, ii) the monopolist chooses price and iii) consumers consume. Since the policy maker does not observe c_t, the equilibrium tax will be a function of only the prior period's price.

The Monopolist

We begin with the monopolist's problem, given τ_t. The monopolist's Bellman equation is:

$$V(p_{t-1}, c_t, \tau_t) = \max_{p_t}\{(p_t - c_t - \tau_t)X_t(p_t, p_{t-1}) + \beta E_t V(p_t, c_{t+1}, \tau_{t+1})\}.$$

To solve this problem, we must form a hypothesis about its expectations over the government's tax policy. In particular, suppose that it is linear in the prior period price:

$$\tau_t = q + rp_{t-1}$$

where q and r are constants. This supposition will be confirmed once we solve the government's optimal tax problem.

To solve the monopolist's problem, we substitute the policy maker's expected tax policy. However, this results in the problem that our solution to the monopolist's profit maximization problem will no longer depend on the current tax. For this reason, we introduce a term representing the policy maker's one period deviation, v_t, from the equilibrium tax. That is,

$$\tau_t = q + rp_{t-1} + v_t.$$

The monopolist's optimal response to v_t is equivalent to its response to τ_t (i.e., $\partial p_t/\partial \tau_t = \partial p_t/\partial \tau_t$) which will be all we need to solve the government's optimal tax problem. In equilibrium there are no deviations so that $v_t = 0$ for all t.

Monopolist's Euler equation is:

$$(p_t - c_t - \tau_t)\frac{\partial X_t^i}{\partial p_t} + X_t^i + \beta E_t \left[V_p^i(p_t, c_{t+1}, \tau_{t+1}) + V_\tau^i(p_t, c_{t+1}, \tau_{t+1})\frac{\partial \tau_{t+1}}{\partial p_t}\right] =$$
$$\hat{a}(1-\beta r) + (\hat{b} - \beta\hat{d})q - \beta\hat{d}\bar{c} + \hat{b}c_t + \hat{b}v_t + \beta(r\hat{b} + \hat{d})p_{t+1} - 2(\hat{b} + \beta r\hat{d})p_t + (r\hat{b} + \hat{d})p_{t-1} = 0.$$

Using methods similar to those at the beginning of the prior section, this has solution:

$$p_t = (1-\lambda)\bar{p} + \lambda p_{t-1} + \frac{\lambda\hat{b}}{r\hat{b} + \hat{d}}(c_t - \bar{c}) + \frac{\lambda\hat{b}}{r\hat{b} + \hat{d}}v_t$$

where

$$\bar{p} = \frac{(1-\beta r)\hat{a} + (\hat{b} - \beta\hat{d})(\bar{c} + q)}{2(\hat{b} + \beta r\hat{d}) - (1+\beta)(r\hat{b} + \hat{d})}$$

and

$$\lambda = \begin{cases} \dfrac{\frac{\hat{b}+\beta r\hat{d}}{r\hat{b}+\hat{d}} - \sqrt{\left(\frac{\hat{b}+\beta r\hat{d}}{r\hat{b}+\hat{d}}\right)^2 - \beta}}{\beta} & \text{if } r\hat{b} + \hat{d} > 0 \\ \dfrac{\frac{\hat{b}+\beta r\hat{d}}{r\hat{b}+\hat{d}} + \sqrt{\left(\frac{\hat{b}+\beta r\hat{d}}{r\hat{b}+\hat{d}}\right)^2 - \beta}}{\beta} & \text{if } r\hat{b} + \hat{d} < 0 \end{cases}$$

Let $m = \partial p_t/\partial \tau_t = \lambda\hat{b}/(r\hat{b} + \hat{d})$.

The Government

The government's problem is to maximize total discounted expected revenues. The Bellman equation for the government is therefore:

$$R(p_{t-1}) = \max_{\tau_t} E_t[\tau_t X_t(p_t, p_{t-1}) + \eta R(p_t)]$$

where η is the government's discount factor. The government's Euler equation is therefore:

$$E_t\left[\tau_t \frac{\partial X_t}{\partial p_t}\frac{\partial p_t}{\partial \tau_t} + X_t + \eta R'(p_t)\frac{\partial p_t}{\partial \tau_t}\right] = 0.$$

Substituting for τ_{t+1}, X_t, and p_t and then solving for τ_t yields:

$$\tau_t = \frac{\hat{a} + \eta q(\hat{d} - \hat{b}\lambda)m}{\hat{b}m} - \frac{[(1 + \eta r\lambda m)\hat{b} - \eta rm\hat{d}](1-\lambda)\bar{p}}{\hat{b}m} + \frac{(1+\eta rm\lambda)(\hat{d} - \hat{b}\lambda)}{\hat{b}m}p_{t-1}$$

In other words, the solution to the government's problem is linear in p_{t-1}, confirming our earlier assumption.

Solving for the coefficient r yields

$$r = \frac{\hat{d}(\hat{d} - \hat{b}\lambda)}{\hat{b}[(2+\eta\lambda^2)\hat{b}\lambda - (1+\eta\lambda^2)\hat{d}]}$$

Note that for $\hat{d} > 0$ ($\hat{d} < 0$), r is strictly decreasing (increasing) in λ and is bounded below (above) by $\hat{d}(\hat{d} - \hat{b})/(\hat{b}[\hat{b} + (1+\eta)(\hat{b} - \hat{d})])$ and above (below) by $\hat{d}/\hat{b}$.