Multi-Period Competition with Switching Costs:
An Overlapping Generations Formulation

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Abstract

I examine an infinite-period duoply market with positive consumer switching costs and overlapping generations of consumers. When consumers have a finite time-horizon, then, unlike Beggs and Klemperer [1992], the two firms may alternate dominance from one period to the next, alternately charging high and low prices. This agrees with the intuition that firms with a high locked-in market share may set price so as to exploit that market share, which causes a subsequent low market share among the new cohort of buyers, leading to lower prices, etc.
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I Introduction

In many markets, consumers (or wholesalers) incur switching costs when changing from one supplier to another. Switching costs can include learning, transaction, and information costs.[1] In addition, they can also include artificial switching costs, created by producers to lock in consumers (e.g., frequent flyer programs or coupons included with packaging).

There have been several papers examining duopoly competition with positive consumer switching costs. Examples of two-period models include Beggs [1989], Caminal and Matutes [1990], and Klemperer [1987a,1987b]. While such models are useful for demonstrating the non-competitive effects of switching costs, their finite horizon renders them incapable of describing how market shares evolve over time. In addition, two-period models may not be entirely satisfactory for considering entry decisions or the effects of exogenous shocks. Hence it is important to consider infinite-horizon models of switching costs.

There are several recent contributions that examine the evolution of market shares and prices in infinite-horizon models of switching costs. For example, Farrell and Shapiro [1988] use an overlapping generations model to get the extreme result that firms alternate between selling to all of the new consumers and selling to all of the old consumers. Also using an overlapping generations model, Padilla [1992] gets a similar result. These extreme, all-or-nothing results are unrealistic and are driven by the assumption of perfect substitutability between goods produced by different firms.

Beggs and Klemperer [1992] instead use a model with imperfect substitutability and where consumers have an infinite horizon. One of their main results is that while firms sell to some of both the young and the old consumers, market shares evolve monotonically. Although they no longer get an all-or-nothing result, a formulation where consumers have a finite horizon is more realistic since people do have finite lives. In addition, even if buyers' time horizons are considered to be infinite (as would be the case if the buyers are wholesalers), a model with a shorter buyer time horizon may be relevant for product classes where each 'generation' of a product is useful only for a limited period. For example, at one time cassettes tapes were the main medium for the storage of data on personal computers. Tapes were superseded by floppy diskettes and hard disk drives, which in turn were superseded by diskettes and hard drives of successively greater capacity. Eventually these storage devices may themselves be replaced by more sophisticated technologies (e.g., CD-RAM drives). While each generation of the product serves the same function, they are very different pieces of hardware, with their own associated learning, installation, and transaction costs. Hence, even if consumers have relatively long time horizons, their problems may be decomposed to shorter horizon maximization problems for each generation of a product.

The alternative to buyers with infinite horizons is a model with overlapping generations of buyers. Using a model with imperfect substitutability and overlapping generations, I examine the evolution of prices and market shares. When consumers have a finite horizon, firms sell to some of both the young and the old consumers, and prices and market shares converge to the steady state. Although prices and market shares converge in both the infinite and finite horizon cases, whether or not convergence is monotonic depends on the finiteness of the consumer's time horizon. The remaining results of Beggs and Klemperer are unchanged.

II The Model

In each period $t = 1, 2, \ldots$ each firm, $j = 0, 1$, simultaneously chooses a price, $p_{j,t}$, given its expectations about how its choice will affect future profits. Given current prices and their expectations about future prices, consumers then decide which firm to buy from.

Consumers live for two periods. In each period a cohort of young consumers enters the market. At the end of their second period of life, they leave the market. Consumers have a reservation value of r and demand a single unit of the good in each period. Consumers from each cohort are uniformly distributed on the interval, $[0, 1]$, with a transportation cost of one per unit of distance. Inclusion of a parameter for the transportation cost is unnecessary as it only acts as a scale factor. Assume the mass of each cohort is normalized to one. Once a young consumer has bought from a particular firm, it is too costly for that consumer later to switch to another firm. Consumers get a utility of zero in any period in which they do not purchase. The model can be easily modified to incorporate population growth. Similar to Beggs and Klemperer, I also assume that a consumer who does not buy must leave the market.

The two firms, $j = 0, 1$, are located at j, so consumer i's transportation cost when buying from firm 0 is i and when buying from firm 1 is $1 - i$. Firm j's share of locked-in consumers is $\sigma_{j,t}$ in any period $t \geq 0$, where $\sigma_{j,0}$ indicates its initial share. When there is no ambiguity, I omit the time subscript and refer to current price and locked-in market share as p_j and σ_j and future (next period) price and locked-in market share as p'_j and σ'_j. Marginal costs are assumed to be identical and are normalized to zero. As inclusion of asymmetric costs does not significantly alter the results, I assume symmetry here to simplify the computations.

Firms have a discount factor δ_F and maximize discounted profits. Consumers have a discount factor δ_C and maximize discounted utility.

III Results

The solution method is constructive and is similar to the approach used by Beggs and Klemperer. When possible, I use the notation of Beggs and Klemperer. I look for Markov equilibria where the state variable is the current share of locked-in consumers and the equilibrium price functions are linear.

Proposition 1 *There exists $(\underline{r}, \bar{r})$ such that if $\underline{r} \leq r \leq \bar{r}$, then there exists a Markov perfect* *equilibrium in which firms' equilibrium strategies are linear. This is the unique equilibrium in which agents pursue linear strategies and all consumers buy in equilibrium.*

Proof: See appendix.

The lower bound on r is required to ensure that all consumers are willing to buy at the prices derived. The upper bound on r is required to ensure that firms do not have an incentive to deviate from these prices by forgoing all new consumers and extracting all of the surplus from their current customer base. Given that these constraints are satisfied, there is a unique Markov perfect equilibrium where firms use a pricing strategy that is a linear function of their current share of locked-in consumers. Note that this does not rule out the existence of other types of Markov equilibria.

In equilibrium, $p_j = d + e\sigma_j$ where $e > 0$ (see appendix) so that price is increasing in locked-in market share—firms with a high locked-in market share exploit that market share by charging high prices. In addition, since $\sigma_j' = \eta - \mu\sigma_j$, with $\eta, \mu > 0$ (see appendix), it can be seen that the equilibrium share of locked-in consumers behaves cyclically. This is due to the fact that locked-in consumers are valuable for only one period—a firm with a high locked-in market share charges a high price, resulting in a low locked-in market share next period. This agrees with the intuition that if a firm has a high locked-in market share, it sets a high price so as to exploit that market share; but, in the absence of an ability to price discriminate, the high price means a small market share among the new cohort of consumers. This, in turn, leads to a low price, etc.

In order to consider the evolution of market shares (i.e., shares of *total* sales), define $\bar{\sigma}_{j,t+1} = (\sigma_{j,t+1} + \sigma_{j,t})/2$—this is firm j's the share of period t total sales. From this I get an expression for equilibrium market share,

$$\bar{\sigma}_j' = \eta - \mu\bar{\sigma}_j. \tag{1}$$

That is, equilibrium market share also behaves cyclically. This is shown formally in the following proposition.

Proposition 2 *Firm j's period-t market share, $\bar{\sigma}_{j,t}$, evolves as*

$$\bar{\sigma}_{j,t} = \frac{1}{2} + (-\mu)^t \left(\bar{\sigma}_{j,0} - \frac{1}{2}\right) \tag{2}$$

where $\bar{\sigma}_{j,0} = (\sigma_{j,0} + \sigma_{j,-1})/2$ and $\sigma_{j,-1} = \eta/\mu - \sigma_{j,0}/\mu$. Firm j's steady-state market share is $1/2$ and μ is in the interval $[2/3, 1)$, strictly increasing in δ_F, and converges to 1 as $\delta_F \to 1$.

Proposition 2 follows from (7), repeated substitution of (1), and from the proof of Proposition 1.

Since $\mu < 1$, it can be seen that market share converges to 1/2. However, market shares are alternately greater than and less than 1/2; hence, convergence is not monotonic.

Notice also that convergence is much slower than in Beggs and Klemperer. Specifically, μ is *at least* three to five times larger than in Beggs and Klemperer. In fact, as firms become more patient (δ_F increases), convergence becomes slower (μ increases). In particular, as $\delta_F \to 1$, $\mu \to 1$ so that in the limit, market share does not converge at all but alternates between $\sigma_{j,0}$ and $1 - \sigma_{j,0}$.

Appendix

Proof of Proposition 1: The strategy of the proof is as follows. Since I am interested in equilibria with linear pricing strategies, I only consider such pricing strategies. Linear pricing strategies imply that value functions must be quadratic and the future share of locked-in consumers will be linear. First, I compute the coefficients that solve the consumer's maximization problem, satisfy the definition of a value function, and maximize firm profits. Then I find the bounds on r that ensure that all consumers purchase in equilibrium and ensure that firms follow this pricing strategy. Existence and uniqueness of an equilibrium in linear strategies follows.

Suppose that the firms' value and price functions are as follows:

$$\Pi_j(\sigma_j) = k + l\sigma_j + m\sigma_j^2 \tag{3}$$

$$p_j(\sigma_j) = d + e\sigma_j. \tag{4}$$

New consumer demand, ν_j, is determined by equating the marginal young consumer's expected payoff from buying from each firm, using the equilibrium prices (4) and then solving:

$$\nu_j = \frac{1}{2} + \frac{1}{2(1 + \delta_C + \delta_C e)}(p_{j'} - p_j). \tag{5}$$

Now, substitute the equilibrium prices (4) into (5) to get firm j's future market share as a function of current market share:

$$\sigma_j' = \eta - \mu\sigma_j \tag{6}$$

where

$$\eta = \frac{1 + \mu}{2} \tag{7}$$

$$\mu = \frac{e}{1+\delta_C+\delta_C e}. \tag{8}$$

Using (4) and the definition of a value function I get:

$$\Pi_j(\sigma_j) = (d+e\sigma_j)(\sigma_j+\sigma_j') + \delta_F \Pi_j(\sigma_j'). \tag{9}$$

Substituting (3) and (6) and equating the coefficients yields the following equations:

$$k = d\eta + \delta_F(k + l\eta + m\eta^2) \tag{10}$$

$$l = d(1-\mu) + e\eta - \delta_F \mu (l + 2m\eta) \tag{11}$$

$$m = e(1-\mu) + \delta_F m \mu^2. \tag{12}$$

In equilibrium, each firm chooses price to maximize the following:

$$p_j(\sigma_j+\nu_j) + \delta_F \Pi_j(\nu_j) \tag{13}$$

where ν_j is as in (5). The first order condition for this maximization problem is:

$$p_j \frac{\partial \nu_j}{\partial p_j} + (\sigma_j+\nu_j) + \delta_F(l + 2m\nu_j)\frac{\partial \nu_j}{\partial p_j} = 0. \tag{14}$$

Substituting the equilibrium market share (6) for ν_j, solving for p_j and then equating the constants yields:

$$d = 2\eta(1+\delta_C+\delta_C e) - \delta_F(l+2m\eta) \tag{15}$$

$$e = 2(1-\mu)(1+\delta_C+\delta_C e) + 2\delta_F m\mu. \tag{16}$$

With the use of (12) and (16), m and e can be solved as functions of μ:

$$m = \frac{2(1-\mu)^2(1+\delta_C)}{(1-2\delta_F\mu+\delta_F\mu^2) - 2\delta_C(1-\mu)(1-\delta_F\mu^2)} \tag{17}$$

$$e = \frac{2(1-\mu)(1+\delta_C)(1-\delta_F\mu^2)}{(1-2\delta_F\mu+\delta_F\mu^2) - 2\delta_C(1-\mu)(1-\delta_F\mu^2)}. \tag{18}$$

From the solution for e together with (8) and some manipulation, it can be seen that μ must satisfy the following cubic equation:

$$\mu(1 - 2\delta_F\mu + \delta_F\mu^2) - 2(1-\mu)(1-\delta_F\mu^2) = 0. \tag{19}$$

At $\delta_F = 0$, it is easy to see that the unique solution to this equation is $\mu = 2/3$. For $\delta_F > 0$, the following is the unique valid solution (i.e., $|\mu| \leq 1$)[2] to this equation:[3]

$$\mu = \frac{2\sin(\arcsin(\sqrt{\delta_F})/3)}{\sqrt{\delta_F}}. \tag{20}$$

μ is strictly increasing and continuous in δ_F and $\mu \in [2/3, 1)$ with $\mu \to 1$ as $\delta_F \to 1$. Substituting μ into (7) yields the solution for η.

Equations (11) and (15) can now be solved for l and d as functions of η, μ, m, and e. Finally, (10) can be solved for k as a function of η, μ, l, m, d, and e.

In order to ensure that consumers always buy at the above prices, it must be that the reservation value is at least as large as the maximum possible price. That is, $r \geq d + e\sigma_j$ for all σ_j. This implies $\underline{r} = d + e$.

In order to ensure that firms have no incentive to deviate from these prices, it must be that for all σ_j, the gain from deviating and extracting all surplus from their current customer base plus the value of starting with a zero customer base in the following period must be no greater than the payoff for playing the prescribed strategy:

$$\begin{aligned}(r - \sigma_j)\sigma_j + \delta_F\Pi_j(0) &\leq p_j(\sigma_j + \sigma_j') + \delta_F\Pi_j(\sigma_j') \\ &= (d + e\sigma_j)(\sigma_j + \sigma_j') + \delta_F(k + l\sigma_j' + m(\sigma_j')^2).\end{aligned}$$

This holds as long as

$$r \leq (d + e\sigma_j)\left(1 + \frac{\sigma_j'}{\sigma_j}\right) + \sigma_j + \delta_F\frac{\sigma_j'}{\sigma_j}(l + m\sigma_j').$$

Let $\bar{r}$ be the minimum of the right hand side. By differentiating with respect to σ_j and noting that the derivative i) has no real roots, ii) is negative for small σ_j, and iii) is continuous on $[0, 1]$, it can be seen that the right hand side is strictly decreasing in σ_j. Hence $\bar{r}$ is equal to the right hand side with $\sigma_j = 1$:

$$\bar{r} = (d + e)\left(1 + \frac{1-\mu}{2}\right) + 1 + \delta_F\left(\frac{1-\mu}{2}\right)\left(l + m\frac{1-\mu}{2}\right). \tag{21}$$

$\bar{r} > \underline{r}$ as long as $l > 0$ and $m > 0$. The solution for l is

$$l = \frac{2(1-\mu^2)(1+\delta_C)(1-\delta_F)}{(1+\delta_F)(1-2\delta_F\mu+\delta_F\mu^2)-2\delta_C(1-\mu)(1-\delta_F\mu^2)}. \tag{22}$$

By examining the solutions for l and m (equation (17)) and using (19) it is clear that they are both positive.

Finally, the second order condition for the firm's profit maximization problem is:

$$\begin{aligned} 2\frac{\partial \nu_j}{\partial p_j} + 2\delta_F m \left(\frac{\partial \nu_j}{\partial p_j}\right)^2 &= \\ -\frac{(1+\delta_C)(1-\delta_F)}{(1+\delta_C+\delta_C e)^2} &< 0. \end{aligned}$$

∎

Endnotes

1. Detailed descriptions can be found in Klemperer [1987a]. See To [1994] for a discussion of switching costs in an international setting.

2. If $|\mu| > 1$, then $\sigma_j \notin [0,1]$ in finite time.

3. The remaining two roots of this equation are:

$$\mu = -\frac{\sqrt{3\delta_F}\cos(\arcsin(\sqrt{\delta_F})/3)}{\delta_F} - \frac{\sin(\arcsin(\sqrt{\delta_F})/3)}{\sqrt{\delta_F}}$$

and

$$\mu = \frac{\sqrt{3\delta_F}\cos(\arcsin(\sqrt{\delta_F})/3)}{\delta_F} - \frac{\sin(\arcsin(\sqrt{\delta_F})/3)}{\sqrt{\delta_F}}.$$

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