Consumer Switching Costs and Private Information

Erick Elder[*]   Ted To[†]

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Abstract

We consider a standard model of consumer switching costs with demand uncertainty where firms observe private information about demand. Given this private information, each firm forms beliefs over different demand realizations as well as beliefs over the other firm's information. The main result here is that in the first period, if firms observe information suggesting that future demand is likely to be high, they will price aggressively, sacrificing current profits for higher market share and the expectation of higher future profits.

Key words: consumer switching costs, oligopoly theory, private information

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[*]College of Business Administration, University of Arkansas at Little Rock, 2801 S. University Ave., Little Rock, AR 72207–1099, USA. Tel: (501) 569-8879 Fax: (501) 569-8871 E-mail: `emelder@ualr.edu`

[†]Centre for the Study of Globalisation and Regionalisation, University of Warwick, Coventry CV4 7AL, United Kingdom. Tel: +44 1203 572557 Fax: +44 1203 572548 E-mail: `T.To@warwick.ac.uk`

1 Introduction

There has recently been a substantial literature on consumer switching costs and their applications. Examples of infinite horizon models include Beggs and Klemperer (1992), Farrell and Shapiro (1988), Klemperer (1987), To (1996) and von Weizsacker (1984). Switching costs have been applied to consider price wars (Klemperer (1989)) and countercyclical pricing in macroeconomics (Chevalier and Scharfstein (1995)) and recently, there has been a growing literature which examines international trade policy under consumer switching costs (Greaney (1997), Hartigan (1996) and To (1994, 1998)).

The standard switching cost models have a number of problems. One problem is that price wars and dumping phenomena are purely first period effects or a result of the entry of new consumers. In general, one would expect that such activities do not happen only in new markets or with the influx of a large number of new consumers but that price wars or dumping occurs repeatedly as firms strike a balance between current profits and market share based on changing market conditions. Second, as long as firms have symmetric information, even if random cost or demand shocks are introduced into the model, in the long run, market shares will converge to a steady state.

We consider the setting in which firms observe noisy, private signals of demand (perhaps the result of market research). Each firm's private information is correlated with the private information of rival firms and thus firms not only update their own beliefs over demand but they also update their beliefs over the information observed by rival firms. Firms then choose prices given these beliefs. In our model, consumers who purchase from one producer in the first period are 'locked in' so that when a firm observes a signal which suggests that future demand is high, that firm will price aggressively in the hopes of capturing market share. These results suggest that price wars are initiated when both firms anticipate relatively high future demand, in the hopes of establishing a dominant market position. In addition, the fact that prices are a result of privately observed information implies that in a fully dynamic model, market shares would shift back and forth over time as firms seek to take advantage of privately observed information.

2 The Model

In each of two periods $t = 1, 2$ each firm, $i = A, B$, simultaneously chooses a price, p_t^i. Given these prices, consumers purchase from one of the two firms. Firms and consumers have discount factors δ_F and δ_C.

Each of the two firms produces a spatially differentiated product. Firms have no fixed costs and have identical marginal costs which have been normalized to zero. Firms A and B are respectively located at 0 and 1. Firms maximize discounted expected profits.

Consumers are uniformly distributed on the interval $[0, 1]$ and incur a transportation cost of one per unit of distance. In each period, consumers have a reservation value of R and inelastically demand a single unit of the good, produced by either firm. We further assume that once a consumer has purchased from one supplier, it is too costly to switch to another supplier. At the end of period 1, mass $\nu \in (0, 1]$ of uniformly and randomly chosen consumers leave the market and are replaced by new consumers. A consumer that leaves the market in the second period does not incur any costs and gets a second-period payoff of zero. Consumers minimize discounted expected price and transportation costs.

Total demand in each period is given by θ_t. In the first period, θ_1 is known with certainty; in the second period, demand is given by $\theta_2 \in \{\theta_H, \theta_L\}$ with $\theta_H > \theta_L$. Let the probability that second period demand is high be given by μ and the probability that demand is low be given by $1 - \mu$. Assume that in the first period, prior to making their first period pricing decision, each firm observes an independent, noisy signal of second period demand. If the actual state is θ_H then firms observe an H with probability ρ_H and if the actual state is θ_L then firms observe an H with probability ρ_L where $\rho_H > \rho_L$. Let the signal that firm i receives be denoted by $S_i \in \{H, L\}$.

2.1 Updating Beliefs

Probabilities μ, ρ_H and ρ_L are common knowledge to both firms and thus form the basis for their prior and posterior beliefs. Upon observing signal $s \in \{H, L\}$, using Bayes rule, we see that a firm's

posterior beliefs that the state is θ_H are given by:

$$\mu_H^* = \frac{\mu\rho_H}{\mu\rho_H + (1-\mu)\rho_L} \qquad \mu_L^* = \frac{\mu(1-\rho_H)}{\mu(1-\rho_H) + (1-\mu)(1-\rho_L)}. \tag{1}$$

where μ_s^* is defined as the probability that $\theta_2 = \theta_H$ conditional on having observed $S_i = s$.

Now, given signal S_i, firm i believes that firm j observed $S_j = H$ with probability $\rho_s^* = P(S_j = H | S_i = s)$. These probabilities are given by:

$$\rho_H^* = \mu_H^* \rho_H + (1 - \mu_H^*)\rho_L \qquad \rho_L^* = \mu_L^* \rho_H + (1 - \mu_L^*)\rho_L. \tag{2}$$

Finally, given $S_i = s$, firms also form beliefs over the joint probability distribution of θ_2 and S_j.

$$\begin{aligned}
\eta_H^{HH} &= \frac{\mu\rho_H^2}{\mu\rho_H + (1-\mu)\rho_L} & \eta_L^{HH} &= \frac{\mu\rho_H(1-\rho_H)}{\mu(1-\rho_H) + (1-\mu)(1-\rho_L)} \\
\eta_H^{HL} &= \frac{\mu\rho_H(1-\rho_H)}{\mu\rho_H + (1-\mu)\rho_L} & \eta_L^{HL} &= \frac{\mu(1-\rho_H)^2}{\mu(1-\rho_H) + (1-\mu)(1-\rho_L)} \\
\eta_H^{LH} &= \frac{(1-\mu)\rho_L^2}{\mu\rho_H + (1-\mu)\rho_L} & \eta_L^{LH} &= \frac{(1-\mu)\rho_L(1-\rho_L)}{\mu(1-\rho_H) + (1-\mu)(1-\rho_L)} \\
\eta_H^{LL} &= \frac{(1-\mu)\rho_L(1-\rho_L)}{\mu\rho_H + (1-\mu)\rho_L} & \eta_L^{LL} &= \frac{(1-\mu)(1-\rho_L)^2}{\mu(1-\rho_H) + (1-\mu)(1-\rho_L)}
\end{aligned} \tag{3}$$

where $\eta_s^{s's''} = P(\theta = \theta_{s'}, S_j = s'' | S_i = s)$.

With these preliminaries aside, we now turn to solving the model. As is usual for sub-game perfect equilibria, we begin in the second period.

3 The Second Period

In the second period, consumers minimize their second period costs given that they are either locked into some producer or that they are new consumers with no previous ties.

Let $x^* = 1/2 + (p_2^B - p_2^A)/2$. New consumer x buys from firm A if $x < x^*$ and from B otherwise. All old consumers buy from the same producer provided that $p_2^i + q_1^i \leq R$. Thus when $|p_2^B - p_2^A| \leq 1$

and the marginal consumers' total cost is no greater than R, firm i's demand is $\theta_2 q_2^i$ where:

$$q_2^i = \frac{1}{2} + \frac{1-\nu}{2}(2q_1^i - 1) + \frac{\nu}{2}(p_2^j - p_2^i). \tag{4}$$

Firm i's second period profits are:

$$\pi_2^i = \theta_2 p_2^i q_2^i \tag{5}$$

Solving both A and B's maximization problem yields second period prices:

$$p_2^i = \frac{1}{\nu} + \frac{1-\nu}{3\nu}(2q_1^i - 1) \tag{6}$$

Substituting (6) into (4) yields firm i's second period output.

$$q_2^i = \frac{1}{2} + \frac{1-\nu}{6}(2q_1^i - 1) \tag{7}$$

Substituting (6) and (7) into (5) results in second period profits of:

$$\pi_2^i = \frac{\theta_2}{2\nu}\left[1 + \frac{1-\nu}{3}(2q_1^i - 1)\right]^2 \tag{8}$$

4   The First Period

Consumers must decide which firm to purchase from, anticipating the outcome in the second period and knowing that if they are still in the market in the second period, they are 'locked-in.'

Let x^{**} be the consumer that is indifferent between purchasing from firm A and from firm B.

$$p_1^A + x^{**} + \delta_C(1-\nu)(p_2^A + x^{**}) = p_1^B + (1 - x^{**}) + \delta_C(1-\nu)(p_2^B + (1 - x^{**})) \tag{9}$$

Firm A's first period output is $q_1^A = x^{**}$ and B's is $q_1^B = 1 - x^{**}$. Substituting q_1^A for x^{**} and (6) for p_2^i, we solve (9) for A's first period demand q_1^A. Firm B's first period demand is $1 - q_1^A$.

$$q_1^i = \frac{1}{2} + \lambda(p_1^j - p_1^i) \tag{10}$$

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where $\lambda = 3\nu/2(3\nu + \delta_C(1 - \nu)(\nu + 2))$.

Given signal S_i, firm i maximizes expected discounted profits through choice of first period prices, knowing how this choice will affect profits in the future. Firm i's discounted expected profits are:

$$E_s[\pi^i] = E_s[\pi_1^i + \delta_F \pi_2^i] \tag{11}$$

Differentiating (11), yields firm i's first order condition:

$$E_s\left[\theta_1\left(q_1^i + p_1^i \frac{\partial q_1^i}{\partial p_1^i}\right) + \delta_F \theta_2 \frac{\partial \pi_2^i}{\partial q_1^i}\frac{\partial q_1^i}{\partial p_1^i}\right] = 0 \tag{12}$$

where π_2^i is as given in (8).

Letting $\alpha = 2\lambda(1 - \nu)/3\nu$ and $\bar{\theta}_{2s} = E[\theta_2]$, the first-order condition for the firm's first period problem can be rewritten as:

$$\theta_1\left(\frac{1}{2} + \lambda E_s[p_1^j] - 2\lambda p_{1s}^i\right) - \delta_F \alpha \bar{\theta}_{2s} + \delta_F \alpha^2 \nu \bar{\theta}_{2s} p_{1s}^i - \delta_F \alpha^2 \nu E_s[\theta_{2s} p_1^j] = 0 \tag{13}$$

The second-order conditions for profit maximization requires that

$$2\theta_1 \lambda > \delta_F \alpha^2 \nu \bar{\theta}_{2s} \tag{14}$$

Solving (13) for p_{1s}^i yields firm i's best response, given its expectations over firm j's prices.

$$p_{1s}^i = \frac{\theta_1/2 + \theta_1 \lambda E_s[p_1^j] - \delta_F \alpha \bar{\theta}_{2s} - \delta_F \alpha^2 \nu E_s[\theta_2 p_1^j]}{2\theta_1 \lambda - \delta_F \alpha^2 \nu \bar{\theta}_{2s}} \tag{15}$$

Given firm i's beliefs over the information of firm j, we can write firm i expectation over the price that firm j will charge as $E_s[p_1^j] = \rho_s^* p_{1H}^j + (1 - \rho_s^*)p_{1L}^j$ where $p_{1s'}^j$ is firm j's first period price contingent on having observed information $S_j = s'$ and firm i, having observed $S_i = s$, believes with probability ρ_s^* that firm j observed $S_j = H$. Similarly, using the conditional joint probability distribution of θ and S_j (i.e., $\eta_s^{s's''}$), $E_s[\theta p_1^j]$ as given by $E_s[\theta p_1^j] = (\eta_s^{HH}\theta_H + \eta_s^{LH}\theta_L)p_{1H}^j + (\eta_s^{HL}\theta_H +$

$\eta_s^{LL}\theta_L)p_{1L}^j$.

Equation (15) must be satisfied for $i = A, B$, $j \neq i$, and $s^i = H, L$ and thus the equilibrium signal contingent prices are given by the solution to a system of four equations in four unknowns. Since marginal costs are the same for each firm, we must have $p_{1s}^i = p_{1s}^j = p_{1s}$ for $s = H, L$, and hence can reduce this system to two equations in two unknowns, p_{1H}, and p_{1L}. This is given by

$$
\begin{bmatrix} p_{1H} \\ p_{1L} \end{bmatrix} = \begin{bmatrix} \dfrac{\theta_1 - 2\delta_F \alpha \bar{\theta}_{2H}}{2D_H} \\ \dfrac{\theta_1 - 2\delta_F \alpha \bar{\theta}_{2L}}{2D_L} \end{bmatrix} + \begin{bmatrix} \dfrac{\gamma_1}{D_H} & \dfrac{\gamma_2}{D_H} \\ \dfrac{\gamma_3}{D_L} & \dfrac{\gamma_4}{D_L} \end{bmatrix} \begin{bmatrix} p_{1H} \\ p_{1L} \end{bmatrix}
\tag{16}
$$

where $\gamma_1 = \theta_1 \lambda \rho_H^* - \delta_F \alpha^2 \nu (\eta_H^{HH}\theta_H + \eta_H^{LH}\theta_L)$, $\gamma_2 = \theta_1 \lambda (1 - \rho_H^*) - \delta_F \alpha^2 \nu (\eta_H^{HL}\theta_H + \eta_H^{LL}\theta_L)$, $\gamma_3 = \theta_1 \lambda \rho_L^* - \delta_F \alpha^2 \nu (\eta_L^{HH}\theta_H + \eta_L^{LH}\theta_L)$, $\gamma_4 = \theta_1 \lambda (1 - \rho_L^*) - \delta_F \alpha^2 \nu (\eta_L^{HL}\theta_H + \eta_L^{LL}\theta_L)$, and $D_s = 2\theta_1 \lambda - \delta_F \alpha^2 \nu \bar{\theta}_{2s} > 0$.

This can be rearranged and then solved using Cramer's Rule and has solution

$$
p_{1H} = \frac{\left(\dfrac{\theta_1 - 2\delta_F \alpha \bar{\theta}_{2H}}{2D_H}\right)\left(\dfrac{D_L - \gamma_4}{D_L}\right) + \left(\dfrac{\theta_1 - 2\delta_F \alpha \bar{\theta}_{2L}}{2D_L}\right)\left(\dfrac{\gamma_2}{D_H}\right)}{D}
\tag{17}
$$

$$
p_{1L} = \frac{\left(\dfrac{\theta_1 - 2\delta_F \alpha \bar{\theta}_{2H}}{2D_H}\right)\left(\dfrac{\gamma_3}{D_L}\right) + \left(\dfrac{\theta_1 - 2\delta_F \alpha \bar{\theta}_{2L}}{2D_L}\right)\left(\dfrac{D_H - \gamma_1}{D_H}\right)}{D}
\tag{18}
$$

where $D = ((D_H - \gamma_1)(D_L - \gamma_4) - \gamma_2\gamma_3]/D_H D_L$.

Proposition 1 *For any admissible parameters, $p_{1H} < p_{1L}$.*

Proof: Calculate $p_{1H} - p_{1L}$.

$$
\begin{aligned}
p_{1H} - p_{1L} &= \frac{\left(\dfrac{\theta_1 - 2\alpha \bar{\theta}_{2H}}{2D_H}\right)\left(\dfrac{D_L - \gamma_4 - \gamma_3}{D_L}\right) - \left(\dfrac{\theta_1 - 2\alpha \bar{\theta}_{2L}}{2D_L}\right)\left(\dfrac{D_H - \gamma_1 - \gamma_2}{D_H}\right)}{\dfrac{(D_H - \gamma_1)(D_L - \gamma_4) - \gamma_2\gamma_3}{D_H D_L}} \\
&= \frac{(\theta_1 - 2\delta_F \alpha \bar{\theta}_{2H})(D_L - \gamma_4 - \gamma_3) - (\theta_1 - 2\delta_F \alpha \bar{\theta}_{2L})(D_H - \gamma_1 - \gamma_2)}{2\left[(D_H - \gamma_1)(D_L - \gamma_4) - \gamma_2\gamma_3\right]}
\end{aligned}
\tag{19}
$$

First, it can be shown that the numerator is strictly positive for any admissible parameter values. Note that $\gamma_1 + \gamma_2 = \theta_1 \lambda - \delta_F \alpha^2 \nu \bar{\theta}_{2H}$ and $\gamma_3 + \gamma_4 = \theta_1 \lambda - \delta_F \alpha^2 \nu \bar{\theta}_{2L}$ then $D_H - (\gamma_1 + \gamma_2) = \theta_1 \lambda$,

and $D_L - (\gamma_3 + \gamma_4) = \theta_1\lambda$. It is straightforward to see that $(\theta_1 - 2\delta_F\alpha\bar{\theta}_{2H})(D_L - \gamma_4 - \gamma_3) - (\theta_1 - 2\delta_F\alpha\bar{\theta}_{2L})(D_H - \gamma_1 - \gamma_2) = -2\delta_F\alpha\theta_1\lambda(\bar{\theta}_{2H} - \bar{\theta}_{2L}) < 0$. It remains to be shown that for any combination of parameters, which satisfy the second-order conditions, that $(D_H - \gamma_1)(D_L - \gamma_4) - \gamma_2\gamma_3 > 0$. Expanding the above results in $D_H D_L - D_H\gamma_4 - D_L\gamma_1 + \gamma_4\gamma_1 - \gamma_2\gamma_3$. After substituting and substantial manipulation, this is equal to: $\lambda\theta_1[2\lambda\theta_1 - \delta_F\alpha^2\nu(\eta_H^{HL}\theta_H + \eta_H^{LL}\theta_L) - \delta_F\alpha^2\nu(\eta_L^{HH}\theta_H + \eta_L^{LH}\theta_L) - \lambda\theta_1(\rho_H^* - \rho_L^*)]$. Since $2\lambda\theta_1 > \delta_F\alpha^2\nu\bar{\theta}_H$, this is greater than:

$$\frac{\lambda^2\theta_1^2}{\bar{\theta}_H}(2\bar{\theta}_H - 2(\eta_L^{HH}\theta_H + \eta_H^{HL}\theta_H + \eta_L^{LH}\theta_L + \eta_H^{LL}\theta_L) + \rho_L^*\bar{\theta}_H - \rho_H^*\bar{\theta}_H)$$
$$= \frac{\lambda^2\theta_1^2}{\bar{\theta}_H}\frac{\mu(1-\mu)(\rho_H - \rho_L)[\rho_H\rho_L(\theta_H - \theta_L) + (\rho_H\theta_H - \rho_L\theta_L)(\mu\rho_H + (1-\mu)\rho_L)]}{[\mu\rho_H + (1-\mu)\rho_L]^2[\mu(1-\rho_H) + (1-\mu)(1-\rho_L)]} \quad (20)$$

Since $\rho_H > \rho_L$ and $\theta_H > \theta_L$, this is always positive. Therefore $p_{1H} < p_{1L}$. ■

In other words, when firms observe information suggesting that second-period demand is going to be high, they price more aggressively than when they observe a signal of low future demand. Price wars are now predicted as a coincidence of privately observed information (i.e., when both firms observe signal H). This occurs with positive probability (i.e., $\mu\rho_H^2 + (1 - \mu)(1 - \rho_L)^2$). Depending on the parameters, this can explain the relative frequency with which price wars are sometimes observed. In a fully dynamic model, one would thus expect that price wars under consumer switching costs do not always need to occur at the inception of a new market or with a large influx of fresh consumers.

Furthermore, since equilibrium prices are contingent on privately observed information, so that even though *ex ante* both firms are symmetric, *ex post* they behave differently with positive probability (i.e., $2(\mu\rho_H(1 - \rho_H) + (1 - \mu)\rho_L(1 - \rho_L))$). Again in a fully dynamic setting, one would expect that prices and market shares would not typically converge to a steady state.

5   Concluding Remarks

We introduce asymmetric private information into a standard consumer switching cost model in order to provide a plausible framework under which price wars can arise as an ongoing phenomenon,

rather than as a first period result as predicted by typical models of consumer switching costs. Although the present model is not fully satisfactory for this purpose, it represents a step in the right direction. To be more satisfactory, one would want to look at a model with a longer time horizon. Unfortunately, fully dynamic models without private information are already quite complicated (e.g., Beggs and Klemperer (1992) and To (1996)). With currently available methods, fully dynamic models with private information are intractable. A simpler model with *pseudo*-dynamics[1] can be solved, however, the solution is algebraically meaningless. Numerical simulations do however indicate that similar results pertain, confirming our conjectures.

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[1]This model is identical to that specified above except that in the first period, rather than being a new market, firms have exogenously specified existing market shares q_0^A and q_0^B.

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