Strategic Resource Extraction, Capital Accumulation and Overlapping Generations ⋆

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Abstract

The standard resource extraction framework assumes infinitely lived agents and yields an overfishing result. For some applications, a finite time horizon may be more appropriate. A direct extension of the Levhari-Mirman model to overlapping generations yields an extreme overfishing result. Alternatively, we assume young and old specialize and respectively fish and supply capital. In this model, under some circumstances there may well be under-utilization of natural resources. However, for a given production technology, if there are a sufficiently large number of agents, overfishing always results.

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1 Introduction

Levhari and Mirman (1980) consider the extraction of a commonly owned, renewable resource in a dynamic setting. Infinitely lived agents decide how

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much of the resource to extract in each period, taking into account, the rate at which the resource regenerates itself and the extraction decisions of other agents. That is, they consider fishing as a differential game or in other words, a dynamic version of the "tragedy of the commons." This original framework has been extended in various ways. [1] One potential drawback of these studies is the assumption that agents are infinitely lived. In some applications, a model where agents have a finite time horizon may be more reasonable. It is well known that the assumption of a finitely-lived agent, or overlapping generations models, may lead to Pareto inefficiencies through the over-accumulation of capital—or in our context the under-exploitation of the resource. It is our purpose, in this paper, examine the robustness of the dynamic tragedy of the commons in the face of the assumption of finitely lived agents and in particular the relationship between the standard under-exploitation externality and the dynamic commons tragedy of over-exploitation.

A simple alternative is to use an overlapping generations framework. However, directly extending the Levhari and Mirman (1980) model to overlapping generations yields the result that the resource are fully extracted in the first period, this is the traditional tragedy of the commons. To see this, notice that the old have no future to look forward to and, therefore, consuming the entire resource stock is optimal for the old. Furthermore, given that the young can expect the old to consume the entire stock, the young also extracts as much of the resource as possible. Although this result is extreme, it has some appeal; agents with little or no future exploit the resource to the greatest extent possible. Rapid over-exploitation, such as suggested by this simple overlapping generations framework, has been seen repeatedly as over-harvesting has endangered many plant and animal species as well as brought about the outright extinction of some species. [2]

Nevertheless, many renewable natural resources do not face such extreme degrees of exploitation. One reason may be because there are technological or legal barriers limiting how much of the resource can be extracted at any moment. For example, given the vastness of the resource, it would be difficult to completely foul the world's supply of clean air in a short period of time. Alternatively, it may be that there is a generational separation between extraction and capital accumulation. We consider the latter alternative and show that if the young extract the resource and the old supply capital then the young may refrain from fully exploiting the natural resource. [3] Moreover, in contrast

[1] See for example, Cave (1987), Datta and Mirman (1999, 2000), Fischer and Mirman (1992, 1996), Hannesson (1997) and Levhari et al. (1981). These extensions are all games with the exception of Levhari et al. which is competitive.

[2] For example, discovered in 1741, Steller's Sea Cow (from the same order as the American Manatee) was hunted to extinction in less than thirty years.

[3] Overlapping generations models are used as metaphors for the finiteness of life spans and the assumption of two-period lives is made for tractability. In a more

to the Levhari and Mirman overfishing result, under the current framework, there may well be underfishing. Nevertheless, for a given technology, if there are enough agents, overfishing results.

2 The Model

In each period there are two generations, a young generation and an old generation. Each individual lives for two periods.

Let X_t be the stock of the renewable natural resource at time t. The rule governing the rate the resource renews itself is given by,

$$X_{t+1} = f(X_t - Y_t) \tag{1}$$

where f is increasing and concave and total extraction in period t is given by Y_t.

Suppose that only the young can extract the resource, part of which is either saved as capital or used as an input for current consumption. The old own capital that is used, in conjunction with the resource, to produce the consumption good. Given this structure, there can now be an incentive for the young to save some of the natural resource. By not extracting all of the resource, the young ensure that they can consume when old.

In each period, the young first decide how much of the natural resource to extract and save as capital, k^i_{t+1}, and how much to extract and use for current production, z^i_t. Total extraction is therefore $y^i_t = z^i_t + k^i_{t+1}$. Following the extraction decision, a competitive market for k and z opens. The current young hold z^i_t and the current old hold k^i_t, which they then trade on this market. Both old and young use capital, k, and the natural resource, z, to produce the consumption good, c, using a homogeneous of degree 1 production function $G(k, z)$.

Note that while trade in the resource and capital are perfectly competitive, the young recognize that their resource extraction decision affects both the price of the fish that they take to market and the price of capital that they take to market once old. To see this, observe that the resource extraction problem is separate from the competitive markets they subsequently face, both when

general framework where agents live for many periods, there will still be an incentive for conservation as long as at some advanced age, individuals are no longer able to fish and must subsist on their savings. Analytic solutions to such models typically require computational methods that are beyond the scope of the current paper (see for example, Auerbach and Kotlikoff, 1987).

young and when old. One can think of the result of the competitive markets as determining a demand function for "resource extraction services." Given this demand for resource extraction services, young agents play a Cournot resource extraction and capital accumulation game with one another.

Assume that each generation has n individuals. If each individual extracts y_t^i for $i = 1, 2, \ldots n$ then $Y_t = \sum_{i=1}^n y_t^i$. Similarly, define z_t^i as the amount of resource young consumer i puts on the market and k_t^i as the amount of capital old consumer i holds so that $Z_t = \sum_{i=1}^n z_t^i$ and $K_t = \sum_{i=1}^n k_t^i$.

Consumers utility is given by,

$$u(c_t^{iy}) + u(c_{t+1}^{io}), \tag{2}$$

where u is increasing and concave. The purchases of k and z by both the young and old are used to produce the consumption good c, using the production function G.

3 Equilibrium

Suppose that $u(c) = \ln c$, $f(X) = X^\alpha$ and $G(k, z) = k^\beta z^{1-\beta}$. The steady state level of the natural resource when there is no extraction is $X = 1$.

In each period, agents play a two-stage game. In the first stage, young consumers extract the natural resource for current consumption or to save as capital. In the second stage consumers purchase k and z to produce the consumption good. As is usual in such settings we begin with the second stage.

3.1 Competitive Marketplace

An old consumer has capital k_t^i and maximizes consumption when old by solving:

$$\begin{aligned} &\max_{\{k_t^{io}, z_t^{io}\}} G(k_t^{io}, z_t^{io}) \\ &\text{subject to: } p_t k_t^{io} + z_t^{io} \leq p_t k_t^i, \end{aligned} \tag{3}$$

where p_t is the price of k relative to the numéraire good z. Since production of the consumption good is Cobb-Douglas the old consumer simply retains a fraction, β, of the endowment, k_t^i:

$$k_t^{io} = \beta k_t^i \tag{4}$$

and sells the remainder at relative price p_t to purchase the resource, which is the numéraire,

$$z_t^{io} = (1-\beta) p_t k_t^i. \tag{5}$$

A young consumer has z_t^i units of the resource and maximizes consumption when young by solving:

$$\max_{\{k_t^{iy}, z_t^{iy}\}} G(k_t^{iy}, z_t^{iy}) \quad \text{subject to: } p_t k_t^{iy} + z_t^{iy} \leq z_t^i. \tag{6}$$

For Cobb-Douglas production of the consumption good, the old consumer purchases k_t^{iy} at price p_t, using the fraction β of the endowment, z_t^i:

$$k_t^{iy} = \frac{\beta z_t^i}{p_t}. \tag{7}$$

The remaining fraction $1-\beta$ of z_t^i is retained,

$$z_t^{iy} = (1-\beta) z_t^i. \tag{8}$$

Total demand for k and z are $\sum_{i=1}^n (k_t^{iy} + k_t^{io})$ and $\sum_{i=1}^n (z_t^{iy} + z_t^{io})$, respectively. Similarly, total supplies are K_t and Z_t as defined earlier. Market clearing therefore requires that,

$$\beta \left(\frac{Z_t}{p_t} + K_t \right) = K_t, \tag{9}$$

and

$$(1-\beta)(Z_t + p_t K_t) = Z_t. \tag{10}$$

This yields the equilibrium price ratio,

$$p_t = \frac{\beta}{1-\beta} \frac{Z_t}{K_t}. \tag{11}$$

3.2 Strategic (Cournot) Resource Extraction

Substituting the equilibrium demands and prices into the young agent's utility function,

$$\begin{aligned} \ln c_t^{iy} + \ln c_{t+1}^{io} &= \ln (k_t^{iy})^{\beta} (z_t^{iy})^{1-\beta} + \ln (k_{t+1}^{io})^{\beta} (z_{t+1}^{io})^{1-\beta} \\ &= \ln \left((1-\beta) \frac{K_t}{Z_t} z_t^i \right)^{\beta} ((1-\beta) z_t^i)^{1-\beta} + \ln (\beta k_{t+1}^i)^{\beta} \left(\beta \frac{Z_{t+1}}{K_{t+1}} k_{t+1}^i \right)^{1-\beta} \\ &= \ln (1-\beta) \frac{{K_t}^{\beta}}{{Z_t}^{\beta}} z_t^i + \ln \beta \frac{{Z_{t+1}}^{1-\beta}}{{K_{t+1}}^{1-\beta}} k_{t+1}^i. \end{aligned} \tag{12}$$

The first stage maximization problem for the young consumers can thus be written as,

$$\begin{aligned}\max_{\{z_t^i, k_{t+1}^i\}} \ln(1-\beta)\frac{{K_t}^{\beta}}{{Z_t}^{\beta}} z_t^i + \ln \beta \frac{{Z_{t+1}}^{1-\beta}}{{K_{t+1}}^{1-\beta}} k_{t+1}^i \\ \text{subject to: } X_{t+1} = \left(X_t - \sum_{i=1}^n y_t^i\right)^{\alpha} \\ Z_{t+1} = Z_{t+1}(K_{t+1}, X_{t+1}),\end{aligned} \tag{13}$$

where $Z_{t+1}(K_{t+1}, X_{t+1})$ is the anticipated equilibrium value of Z_{t+1} and (K_{t+1}, X_{t+1}) is the $t+1$ period state.

To solve this problem, we hypothesize that for Cobb-Douglas production functions the equilibrium value of Z_{t+1} is a constant fraction of X_{t+1} and is independent of K_{t+1}. That is, let $Z_{t+1} = \eta X_{t+1}$. Once the model has been solved, these hypotheses are shown to be consistent with the equilibrium solutions and are therefore justified.

Notice that maximizing the young consumer's objective function with respect to z_t^i and k_{t+1}^i is equivalent to substituting $k_{t+1}^i = y_t^i - z_t^i$ and maximizing with respect to z_t^i and y_t^i. Substituting $Z_{t+1} = \eta(X_t - Y_t)^{\alpha}$ and then taking the first order conditions with respect to z_t^i and y_t^i yields:

$$-\frac{\beta}{Z_t} + \frac{1}{z_t^i} + \frac{1-\beta}{Y_t - Z_t} - \frac{1}{y_t^i - z_t^i} = 0, \tag{14}$$

$$-\frac{\alpha(1-\beta)}{X_t - Y_t} - \frac{1-\beta}{Y_t - Z_t} + \frac{1}{y_t^i - z_t^i} = 0. \tag{15}$$

In a symmetric Nash equilibrium $y_t^i = y_t^j$ and $z_t^i = z_t^j$ for any $i, j = 1, 2, \ldots n$. Solving (14) yields:

$$Z_t = \frac{n-\beta}{2n-1} Y_t. \tag{16}$$

Since $Z_t + K_{t+1} = Y_t$, solving for K_{t+1} yields:

$$K_{t+1} = \frac{n-(1-\beta)}{2n-1} Y_t. \tag{17}$$

Substituting K_{t+1} into (15) and solving for Y_t yields:

$$Y_t = \frac{2n-1}{(2n-1)+\alpha(1-\beta)} X_t. \tag{18}$$

The equilibrium level of extraction is a constant fraction of X_t. This extraction rate is strictly increasing in the number of agents, n, approaching full extraction as n gets large.

Using the solution for Y_t and substituting into (16) and (17) yields:

$$K_{t+1} = \frac{n-(1-\beta)}{(2n-1)+\alpha(1-\beta)}X_t, \tag{19}$$

and

$$Z_t = \frac{n-\beta}{(2n-1)+\alpha(1-\beta)}X_t. \tag{20}$$

This verifies our prior assumption that in equilibrium, Z_t is a constant fraction of X_t.

Finally, substituting Y_t into the law of motion yields,

$$X_{t+1} = \left(\frac{\alpha(1-\beta)}{(2n-1)+\alpha(1-\beta)}X_t\right)^{\alpha}, \tag{21}$$

which has steady state,

$$X_{SS} = \left(\frac{\alpha(1-\beta)}{(2n-1)+\alpha(1-\beta)}\right)^{\frac{\alpha}{1-\alpha}}. \tag{22}$$

It is straightforward to see that X_{SS} is strictly less than one, the natural steady state when there is no extraction.

4 The Golden Rule

The *Golden Rule* used in growth theory refers to the steady state of an economy at which total, per period, consumption is maximized. It is useful here to modify this concept for our model of resource extraction and capital accumulation in order to understand the welfare properties of our equilibrium better.

Our model is different from the standard growth framework in that there are two forms of capital: the commonly owned resource stock, X_t, and the privately owned capital stock, K_t. We first define the Golden Rule individually for each form of capital. The Golden Rule rate of resource extraction must maximize the steady state, per-period, extraction of Y. Similarly, given some steady state rate of resource extraction, Y^*, the Golden Rule rate of capital accumulation maximizes, per-period, consumption. Looking at both forms of capital together, we say that the *Globally Golden Rule* holds if both the Golden Rule rate of resource extraction and the Golden Rule rate of capital accumulation hold.

The Golden Rule level of resource stock maximizes $f(X) - X$ (i.e., if X is the steady-state resource stock remaining after $Y = f(X) - X$ units of the resource has been extracted). Thus, the Golden Rule level of the resource stock must solve $f'(X_{GR}) = 1$. Given X_{GR}, $Y_{GR} = f(X_{GR}) - X_{GR}$.

Given a steady state level of resource extraction, Y^*, the Golden Rule level of capital accumulation is a pair (K, Z) which maximizes total per period consumption.

$$\max_{\{K,Z\}} G(K, Z) \qquad \text{subject to: } K + Z \leq Y^* \tag{23}$$

Since G is homogeneous, K_{GR} is some constant fraction of Y^*, say ϕ.

For our example with Cobb-Douglas production this yields $X_{GR} = \alpha^{\frac{1}{1-\alpha}}$, $Y_{GR} = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)$ and $K_{GR} = \beta Y^*$. A simple comparison of X_{SS} and X_{GR} reveals that if α is relatively small then $X_{SS} > X_{GR}$ and if α is relatively large then $X_{SS} < X_{GR}$. In other words, the steady state rate of resource extraction can be either less than or greater than the Golden Rule rate of resource extraction. As in Diamond (1965), if $X_{SS} > X_{GR}$, the equilibrium rate of resource extraction is Pareto inefficient (i.e., aggregate consumption in at least one period can be increased for both generations by increasing the amount of the resource extracted in that period and then extracting according to the Golden Rule ever after).

Our under-extraction (or equivalently overaccumulation) result bears some similarity to Diamond (1965), however, there are two other factors at work in this model. In addition to the Diamond incentive for overaccumulation, there are two opposing incentives in the extraction of the natural resource. The first is the standard incentive to overexploit a publicly owned resource. The second is an incentive to restrict the supply of the resource to enhance market power. Although the trading of k and z is competitive, endowment levels are determined prior to the opening of the market and, thus, agents can fully anticipate the effect of their extraction decisions on the prices of k and z. This market externality shows up in dynamic economics in which there are long lived agents, see Datta and Mirman (1999, 2000). In other words, young agents play Cournot against one another in making their resource extraction decisions. Since the ability to restrict supplies weakens as the number of agents grows, the incentive to over-extract dominates when n is sufficiently large.

Similarly, it is not surprising that the steady state rate of private capital accumulation is often inefficient. In particular, the rate of private capital accumulation is optimal only for $n = 1$ or $\beta = 1/2$.[4] When $n > 1$, there is too much capital accumulation if $\beta < 1/2$ and too little capital accumulation if

[4] Our results that Golden Rule allocations are not generally attained also hold if young consumers discount old consumption, but the precise conditions under which the Golden Rule is equal to the Cournot Nash equilibrium changes.

$\beta > 1/2$. In this case also, in addition to the Diamond (1965) incentive for overaccumulation, there are the two opposing strategic incentives noted above.

Given that the private rate of capital accumulation satisfies the Golden Rule when $n = 1$, it is useful to calculate the steady state of the public resource stock. In this case:

$$X_{SS} = \left(\frac{\alpha(1-\beta)}{1+\alpha(1-\beta)} \right)^{\frac{\alpha}{1-\alpha}}. \tag{24}$$

Notice that even in this case, where private capital accumulation is efficient, the natural resource may be over- or under-extracted. Thus unlike Levhari and Mirman (1980) where if there is just a single agent extracting the resource, the level of resource extraction is efficient, with overlapping generations, extraction is often Pareto inefficient *and* almost always defies the Golden Rule.

5 Concluding Remarks

As with the original Levhari and Mirman model, in the limit as n gets large, the steady state fish stock goes to zero. This is the traditional tragedy of the commons problem where too many agents using a public resource leads to overuse of that resource. However, as we have demonstrated, there need not be a tragedy. Or more accurately, the tragedy may be that not enough of the resource is consumed. In particular, for fixed n (we can take n as large as we like), if returns-to-scale is declining at a sufficient rate (α is sufficiently small), a combination of the Diamond and strategic incentives leads to too little resource extraction. That is, no matter how many agents there are, there is some parameter configuration for which there is Pareto inefficient under-extraction of the resource.

Since the Globally Golden Rule is generically unattainable, are there any policy prescriptions that would bring it about? Consider in turn three possible cases: i) over-extraction of the resource and overaccumulation of capital, ii) under-extraction of the resource and under-accumulation of capital and iii) either over-extraction of the resource and under-accumulation of capital or under-extraction of the resource and overaccumulation of capital. In case i), the Globally Golden Rule can be straightforwardly attained by setting the appropriate quotas on both resource extraction and capital accumulation.[5] Case ii) is problematic since in an economy with just two goods and no money, corrective policy involving only subsidies cannot be funded. Finally, it may be possible to attain the Globally Golden Rule in case iii). Consider, for example, the case in which there is over-extraction and under-accumulation of capital.

[5] One might also think about a tax policy in this case but quotas are simpler in that there are no proceeds to be distributed.

The relevant policy in this case would be to tax resource extraction, subsidize capital accumulation and impose a quota (possibly non-binding) on capital accumulation. If tax receipts are sufficient, the quota would be binding and the Globally Golden Rule would be attained. To sum up, when there is both under-extraction and under-accumulation, there is little that can be done since there is no way to fund corrective policy. In all other cases, there is scope for corrective policy and quite often, this corrective policy yields Golden Rule allocations.

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