OLIGOPSONY AND THE DISTRIBUTION OF WAGES

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ABSTRACT. We propose a simple model of wage dispersion arising from oligopsonistic competition in the labor market. Our model has workers who are equally able but who have heterogeneous preferences for non-wage characteristics, while employers have heterogeneous productivity characteristics. We completely and explicitly solve for the equilibrium wage distribution and show that "inside" and "outside" forces interact in wage determination. This interaction generates spillover effects of minimum wages in a manner which is consistent with the empirical evidence.

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1. Introduction

This paper presents a simple model of oligopsonistic wage dispersion which is motivated by the empirical evidence on the structure of wages. This evidence presents a challenge to the competitive theory of labor markets, where workers are paid their marginal products, and an individual's wage depends only on individual specific ability. In particular, the empirical evidence finds large inter-industry wage differentials for workers with identical characteristics.[1] Even within industries, there is evidence that wages vary significantly (Dunlop, 1957; Groshen, 1991). There is also evidence that large establishments tend to pay substantial wage premiums (Brown and Medoff, 1989). Moreover, the effect on the wage distribution of an increase in the minimum wage reinforces these puzzles—rather than simply truncating the wage distribution, such a hike often raises wages, for minimum wage workers as well as those who are paid more than the minimum wage (Card and Krueger, 1995; Dolado et al., 1997; Grossman, 1983). Further, a minimum wage produces a "spike" in the distribution of wages (Card and Krueger, 1995).[2]

We present a model of the labor market whose predictions are consistent with the empirical evidence on the distribution of wages. Our model's key predictions are two-fold. First, workers of *identical* ability are paid different wages by different employers and the resulting distribution of wages is consistent with the observed wage distributions. Second, the imposition of a minimum wage raises the wages of minimum wage workers, has a spillover effect on the wages of higher-paid workers while compressing the wage distribution. Finally, our model also predicts that a minimum wage generates a spike in the wage distribution at the minimum. Apart from its explanatory power, a key feature of our model is its tractability and simplicity, which makes it amenable for empirical analysis. While there exist other explanations for wage dispersion such as efficiency wage theory (Albrecht and Vroman, 1998; Bulow and Summers, 1986; Ramaswamy and Rowthorn, 1991; Stiglitz, 1985) and job search models (Burdett and Mortensen, 1998), we believe that our

[1]Slichter (1950) was an early attempt to quantify the degree of wage dispersion and subsequently there have been numerous contributions. See for example Blackburn and Neumark (1992); Dickens and Katz (1987b); Gibbons and Katz (1992); Krueger and Summers (1988); Murphy and Topel (1987).

[2]While modifications of competitive theory can produce wage dispersion (e.g., sorting and compensating differentials), these modifications are limited in their explanatory power (see Katz (1986) and Krueger and Summers (1987) for surveys).

model provides an explanation for these empirical facts, which is, in some ways more persuasive—a comparison of the relative merits of these theories is deferred to Section 4.

Our model relies on two key assumptions to generate these predictions. The first assumption is that workers with *identical skills and abilities* have heterogeneous preferences over non-wage characteristics of employers. These include the actual job specification, hours of work, distance of the firm from the worker's home, the social environment in the workplace, etc. Our approach is one of horizontal job differentiation—we assume that different workers have different preferences over non-wage characteristics.[3] Heterogeneous non-wage preferences ensure that employers have market power in wage setting. That is, we have what is classically referred to as *oligopsony*.[4]

The second important assumption is that the *marginal* product of labor varies between employers—note that this is well consistent with the *average* product of labor or profitability being the same across firms. Indeed, such heterogeneity is unavoidable if firms from different product markets compete in the same labor market. Given employer wage setting power, we show that employer heterogeneity maps into the wage distribution in an interesting way which combines "inside" and "outside" factors in wage setting.

Our main results follow from the solution for the equilibrium wage distribution for arbitrary firm productivities. We find that firms offer different wages in equilibrium with "high productivity" firms typically offering higher wages. However, the wages offered also depend upon outside factors, due to competition in the labor market. The precise pattern of interaction affects the extent of wage dispersion. We also find that firms which offer high wages employ more workers also tend to be more profitable. Finally, a minimum wage causes spillover effects on firms paying above the minimum, reduces wage dispersion and introduces a "spike" in the wage distribution.

[3]The importance of non-wage characteristics has been recognized in the theory of compensating differentials, which is a theory of vertical differentiation. Some jobs are good while other jobs are bad, and wage differentials compensate workers for these differences in characteristics. McCue and Reed (1996) provide survey evidence of horizontal heterogeneity in worker preferences.

[4]The literature on oligopsony is sparse, consisting primarily of empirical evaluations of oligopsony power—see (Boal and Ransom, 1997) for a survey. An early discussion of oligopsony in the context of the market for nurses is Sullivan (1989). There is some discussion of oligopsony in the agricultural economics literature (Chen and Lent, 1992). More recent theoretical treatments include Kaas and Madden (1999) and Naylor (1996).

2. The Model

We now present our model of an imperfectly competitive labor market.[5] Its central feature is that jobs differ in terms of their non-wage characteristics and employers differ in characteristics which affect the marginal revenue product of labor. To model horizontal differentiation in a simple and tractable way, we adapt the influential model of Salop (1979b). We assume that the job characteristic space is a circle of unit circumference. Workers of equal ability are uniformly distributed along all points of the circumference. Let there be n firms in the market. Following Salop, we do not model the location choices of firms, but assume that these firms are uniformly spaced around the circle. A worker who travels distance d to work in a firm incurs a "transportation" cost of td (i.e., this cost is linear in distance, and t is the unit transportation cost). In evaluating wage offers at two firms, a worker takes into account the wages offered as well as the "transport" cost incurred in working at each of these firms. Workers at different locations will evaluate job offers differently, since they will have different transport costs associated with work at any firm.

We allow for a diversity of workers' reservation wages, in the simplest possible way, by assuming that there is a unit mass of low reservation wage workers who are uniformly distributed along the circle, and a mass δ of high reservation wage workers who are similarly uniformly distributed. For simplicity we set the former's reservation wage to zero, and assume that the latter's reservation wage is $v > 0$. Our basic results extend to the general case where we have any arbitrarily large finite set of types of workers at each location.

A worker will choose to work as long as the wage less their transportation cost is at least their reservation wage. Our focus is on parameter values where, in equilibrium, all low reservation wage workers work and only some high reservation wage workers work. This ensures that there is competition for workers between firms and that total employment can vary.

2.1. **Labor Supply.** We consider a model of oligopsony where there is no free entry or exit so that the number of firms, n, is fixed. With n firms in the market, the distance between firms is $1/n$. Suppose that firm i offers wage w_i and one of firm i's nearest rivals, j, is offering

[5]This model has the same basic structure as Bhaskar and To (1999) but differs in two important ways. In Bhaskar and To, we focus on the employment effects of minimum wages. In order to do so, we allow for free entry and exit but restrict employers to have identical productivity characteristics. We elaborate on these differences and briefly discuss the main result of Bhaskar and To in Section 4.

wage w_j. Consider a low reservation wage worker who is located between firms i and j at distance x from i and $1/n - x$ from j. Such a worker will work for firm i if $w_i - tx > w_j - t(1/n - x)$, and will work for i's rival if this inequality is reversed. A worker located at a distance $x^0 \in (0, 1/n)$ is indifferent between working for firm i and i's closest neighbor when:

$$w_i - tx^0 = w_j - t(1/n - x^0).$$

Solving for x^0 we see that $x^0 = (t/n + w_i - w_j)/2t$, provided that $|w_i - w_j| \leq t/n$. Since all workers located up to a distance of x^0 from firm i have lower transportation costs they will work for firm i. Similarly, all workers located farther than x^0 from firm i have higher transportation costs and will work for i's rival. Since there is a similar set of workers on the other side of firm i, firm i's supply of 0-reservation wage workers is

$$\frac{t/n + w_i - \bar{w}_i}{t}$$

where $\bar{w}_i$ is the mean wage offered by i's two nearest neighbors.

Consider now the supply of high reservation wage workers. A high reservation wage worker located at distance x from its most attractive potential employer, firm i, will not work for i if $w_i - tx < v$ but will work for firm i if $w_i - tx > v$. Let $x^v \in (0, 1/n)$ be the distance at which a high reservation wage worker is indifferent between working for firm i and not working at all, i.e., $v = w_i - tx^v$. Solving for x^v yields,

$$x^v = \frac{w_i - v}{t}$$

provided that $w_i \geq v$. Again, all workers located between firm i and x^v work for firm i and those located farther than x^v do not work. Hence firm i's supply of v-reservation wage workers is $2\delta x^v$.

Therefore, whenever $|w_i - w_j| \le t/n$ for $j = i-1, i+1$ and $w_i \ge v$ firm i's total labor supply is:[6,7]

$$L_i = \frac{t/n - 2v\delta + (1+2\delta)w_i - \bar{w}_i}{t}. \tag{1}$$

Equation (1) shows that labor supply is increasing in the firm's own wage, w_i, but decreasing in the average wage paid by its immediate neighbors, $\bar{w}_i = (w_{i-1} + w_{i+1})/2$.[8] However, due to variations in the participation rate (due to the presence of high reservation wage workers), the former effect is larger than the latter, so that a unit increase in both w_i and $\bar{w}_i$ leads to increased labor supply for firm i. This also implies that the elasticity of labor supply for the individual firm exceeds the elasticity of industry labor supply. Thus the situation differs from both monopsony and perfect competition—under monopsony there is no distinction between the two elasticities and under perfect competition, labor supply is infinitely elastic at the level of the firm.

2.2. **Firm Profit Maximization.** We now turn to the firm's output decisions, which affect labor demand. Firm i's output is given by the homogeneous of degree one production function:

$$Y_i = L_i f_i(K_i/L_i) \tag{2}$$

where K_i is i's capital input and f_i is assumed to be twice differentiable, increasing and concave, so that $f_i'' < 0$. Since different workers have identical skills and abilities, they enter the production function uniformly. However, we allow both the production function and (and product price) to be firm-specific, since different firms could be in different industries. Even within the same product market, firms may have different production functions because of firm specific characteristics such as, differing managerial talent, different

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[6]Henceforth, any arithmetic in the indexes of firms, is implicitly $\mathrm{mod} n$, (e.g., $i-1$ represents $(i-1+n) \bmod n$, $i+1$ represents $(i+1) \bmod n$ and $i+j$ represents $i+j \bmod n$).

[7]If workers face linear transportation costs, the firm's profits are neither quasi-concave nor continuous as a function of its wages. In particular, at a wage slightly above $w_i = w_{i+1} + t/n$, the firm will attract all the workers of firm $i+1$, and hence its labor supply jumps discontinuously. We will later derive parameter restrictions which ensure that at the equilibrium wage distribution, the conditions for global optimum are satisfied for every firm.

[8]That is, one firm's wage setting decision has an externality effect on other firms' labor supply. This externality will have an important effect on the equilibrium wage distribution.

production techniques, different access to assets with varying productivities (fertile vs. infertile land), etc.[9]

Firm i's profits can be written as follows:

$$\pi_i = p_i L_i f_i(K_i/L_i) - rK_i - w_i L_i. \tag{3}$$

where p_i is the price of firm i's output and r is the capital rental rate.[10] Product prices may differ due to product differentiation or because firms competing within the same labor market may be selling different goods. Using firm i's first order condition with respect to capital, we can rewrite its profits as

$$\pi_i = \phi_i(p_i, r)L_i - w_i L_i \tag{4}$$

where $\phi_i(p_i, r) = p_i[f_i(k_i(r/p_i)) - f_i'(k_i(r/p_i))k_i(r/p_i)]$, and k_i is the optimal capital-labor ratio, which depends upon (r/p_i). We call ϕ_i firm i's *net revenue product of labor* which differs from its marginal revenue product in that firm i is optimally adjusting its capital labor ratio.

Substituting labor supply (1) into profits (4) and then solving the first order condition yields the firm's optimal wage as a function of the mean wage set by its nearest rivals:

$$w_i = \alpha_i + \beta \bar{w}_i \tag{5}$$

where

$$\alpha_i = \frac{(1+2\delta)\phi_i - t/n + 2\delta v}{2(1+2\delta)} \tag{6}$$

$$\beta = \frac{1}{2(1+2\delta)}. \tag{7}$$

Observe that the individual firm's optimal wage, w_i, is an increasing function of the wage set by other firms, $\bar{w}_i$. This implies that we have a situation of *strategic complementarity* in wage setting. As we shall see later, this has important implications for the effects of minimum wage legislation upon firms which are initially paying wages above the minimum wage, and would therefore seem to be unaffected by such legislation.

[9]Differences in worker productivities does not necessarily drive out firms that are less productive. As it is commonly argued—even with a perfectly competitive product market—assets which lead to higher productivities will command a higher price. Hence firm profits may not be higher in firms with greater productivity.

[10]The fact that labor supply functions are upward sloping guarantees a unique profit maximum.

Equations similar to (5) have been popular in some recent empirical work in labor economics (Abowd and Lemieux, 1993; Blanchflower et al., 1996; Nickell and Wadhwani, 1990, for example). This literature estimates a wage equation where the wage depends upon "inside" factors such as firm profitability and upon a common "outside" wage $\bar{w}$. Our theoretical wage equation has a similar form, since α_i captures the inside factors while β is the coefficient of the outside wage. Note however that the theoretical model says that the outside wage is *firm-specific,* i.e., $\bar{w}_i$. To re-interpret these wage equation estimates, our theory suggests that the outside wage in such empirical work is endogenous so that using a common outside wage introduces measurement error. In particular, the estimated coefficient on the outside wage will in general be biased. Moreover, this implies that the estimated coefficient on inside factors will also be biased. In other words, our model suggests that one should be careful in proxying the outside wage, especially for firms which operate in a spatially separated environment.

3. Equilibrium Wage Dispersion

The optimal wage-setting rule for all firms can be reformulated in matrix notation as follows:

$$\hat{\mathbf{w}} = \boldsymbol{\alpha} + B\mathbf{w} \tag{8}$$

where the matrix B is given by

$$B = \begin{bmatrix} 0 & \frac{\beta}{2} & 0 & \cdots & 0 & \frac{\beta}{2} \\ \frac{\beta}{2} & 0 & \frac{\beta}{2} & & 0 & 0 \\ 0 & \frac{\beta}{2} & 0 & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \frac{\beta}{2} & 0 \\ 0 & 0 & & \frac{\beta}{2} & 0 & \frac{\beta}{2} \\ \frac{\beta}{2} & 0 & \cdots & 0 & \frac{\beta}{2} & 0 \end{bmatrix} \tag{9}$$

Two features of the optimal wage setting rule (8) are noteworthy and have implications for comparative statics. First, optimal wage setting is given by an increasing map, and second, it is a contraction mapping. This implies that a Nash equilibrium exists, is unique and is given by a matrix equation of the form $\mathbf{w}^* = (I - B)^{-1}\boldsymbol{\alpha}$. After solving for $Q = (I - B)^{-1}$, it can be shown that each employer's equilibrium wage is a positive weighted sum of all α_j's where the impact of rival j on firm i's wage declines in j's distance from i. Furthermore, i's own α_i has the greatest weight.

Define $[\,\cdot\,]$ to be the greatest integer function (i.e., $[x] = \max\{i \in \mathbb{I} \mid i \leq x\}$). Denoting a *wage distribution* to be a vector of wages, $\mathbf{w}$, we state the above results more formally,

PROPOSITION 1. *If t, δ and v are sufficiently large then there exists $\alpha^* > \alpha_* > 0$ such that if $\boldsymbol{\alpha} \in [\alpha_*, \alpha^*]^n$ then a Nash equilibrium wage distribution, $\mathbf{w}^*$, exists and is unique and for $i = 0, 1 \ldots n-1$,*

$$w_i^* = \sum_{j=0}^{n-1} q_j \alpha_{i+j} \tag{10}$$

where $q_j = q_{n-j}$ for $j = 1, \ldots [n/2]$ and $q_0 > q_1 > \ldots > q_{[n/2]} > 0$.

Proof. See Appendix.

The matrix Q specifies the mapping from the vector $\boldsymbol{\alpha}$ to the equilibrium wage distribution. This mapping only depends upon δ, the mass of high reservation wage workers, and therefore does not depend upon firm specific characteristics. The vector $\boldsymbol{\alpha}$ depends upon individual firm characteristics, since each component α_i depends upon ϕ_i, net revenue product in firm i—for brevity, we shall refer to this as "productivity" in future. Hence the above proposition completely characterizes the equilibrium wage distribution in terms of the primitives of the model, the firm specific productivity levels as given by the vector $\boldsymbol{\alpha}$. As we will see later, this wage distribution is, as a general rule, non-degenerate.

Since firms in different industries have different production techniques and face different prices, their net revenue products will differ (i.e., $\phi_i \neq \phi_j$ for $i \neq j$). Thus these results are consistent with inter-industry wage differentials. Furthermore, firms within the same industry can have different production functions, and as a result different productivities. Thus our results are consistent with the existence of intra-industry wage differentials. To the extent that prices and production techniques are likely to be more similar within an industry, we expect that measured intra-industry wage differentials should typically be smaller than measured inter-industry wage differentials.

We can exploit this characterization to examine some properties of this dispersed wage equilibrium. We start by defining some additional notation. Given vectors $\mathbf{y}$ and $\mathbf{y}'$, write $\mathbf{y} > \mathbf{y}'$ if $y_i \geq y_i'$ $\forall i$ and $\mathbf{y} \neq \mathbf{y}'$. Write $\mathbf{y} \gg \mathbf{y}'$ if $y_i > y_i'$ $\forall i$. Finally, let $M(\mathbf{y}) = \sum_{i=1}^{n} y_i/n$ denote the mean value of vector $\mathbf{y}$. From Proposition 1, note that an increase in firm i's productivity has a strictly positive effect on the wage of every firm in this economy. Hence, if $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$ are such that $\boldsymbol{\alpha} > \boldsymbol{\alpha}'$, then $\mathbf{w}^* \gg \mathbf{w}^{*\prime}$.

The above results require that the vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$ be ordered, by the partial order $>$. However, one also has results relating the sum of productivities (or average productivity) and the average level of wages, regardless of the distribution. In particular, if $M(\boldsymbol{\alpha}) = M(\boldsymbol{\alpha}')$ then $M(\mathbf{w}^*) = M(\mathbf{w}^{*\prime})$ and if $M(\boldsymbol{\alpha}) > M(\boldsymbol{\alpha}')$ then $M(\mathbf{w}^*) > M(\mathbf{w}^{*\prime})$.

Note that $M(\mathbf{w})$ denotes the mean of wages paid where each firm has the same weight, regardless of its employment level. This can differ from the mean of wages received by workers, since higher wage firms will on average also have higher employment levels. For example, in comparison with the case where all firms have equal productivities at $\boldsymbol{\alpha}$, if $\boldsymbol{\alpha}'$ is a mean preserving spread of $\boldsymbol{\alpha}$, then the mean of wages received will be higher under $\boldsymbol{\alpha}'$ since the higher wages will have a larger employment weight. To see this more generally, notice that for each pair of rival firms, the firm offering a higher wage will have a larger share of the workers between it. Thus the average wage earned by workers is higher. Loosely speaking, the more heterogeneous are employers, the higher are average wages earned by workers within that labor market.

Finally, we note that for given any generic vector $\boldsymbol{\alpha}$, the associated equilibrium wage distribution has the property that no two firms will offer the same wage. In other words, with firm heterogeneity, wage non-uniformity is the norm in our model. This is relevant since it shows how a minimum wage will ensure that some firms end up paying the same wage, thus generating a "spike" in the wage distribution.

To summarize,

PROPOSITION 2. *In the Nash equilibrium,*

i) if $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$ are such that $\boldsymbol{\alpha} > \boldsymbol{\alpha}'$ then $\mathbf{w}^* \gg \mathbf{w}^{*\prime}$,
ii) if $M(\boldsymbol{\alpha}) = M(\boldsymbol{\alpha}')$ then $M(\mathbf{w}^*) = M(\mathbf{w}^{*\prime})$ and if $M(\boldsymbol{\alpha}) > M(\boldsymbol{\alpha}')$ then $M(\mathbf{w}^*) > M(\mathbf{w}^{*\prime})$, and
iii) for almost all $\boldsymbol{\alpha} \in [\alpha_*, \alpha^*]^n$, if $i \neq j$ then $w_i^* \neq w_j^*$.

Proof. See Appendix.

3.1. **Interaction Structure: Inside vs. Outside Factors in Wage Determination.** The main point of this section is to demonstrate that the extent of wage dispersion does not depend solely upon the distribution of productivities, but also upon the interaction structure between firms. More precisely, if $\boldsymbol{\alpha}'$ is a permutation of $\boldsymbol{\alpha}$, this does not imply that the associated wages $\mathbf{w}'$ and $\mathbf{w}$ are permutations of each other. Furthermore, wage dispersion is greater when like competes with like in the labor market, than in the case when firms of one type compete to a greater extent with firms of the other type. In other words, the relative role of inside and outside factors depends upon the pattern of interaction.

To illustrate these points most simply, we shall assume that half the firms are of type H, having high net revenue product, ϕ_H, while the rest are of type L, having low net revenue product, ϕ_L.[11] We shall see that the distribution of wages depends upon the precise pattern of competition between the two types of firm. Suppose that we have k firms of type H arranged contiguously on the circle, followed by k firms of type L, followed by k firms of type H, and so on.[12] The larger the value of k, the more that like interacts with like, while the smaller the value of k, the more that like and unlike interact. We call such an interaction structure a k type interaction structure. The mean level of wages offered, $\bar{w}$, is invariant with respect to k. However, as k increases, we shall see that the wage distribution becomes increasingly unequal—indeed, if we consider two different values of k, the wage distribution when k is smaller second order stochastically dominates the wage distribution when k is larger.

If $k = 1$, the H and L type firms alternate in location. Hence a high type firm's immediate neighbors are of low type, and vice-versa. Nash equilibrium wages are given by:

$$w_H^*(k=1) = \frac{\alpha_H + \beta\alpha_L}{1-\beta^2} \tag{11}$$

$$w_L^*(k=1) = \frac{\alpha_L + \beta\alpha_H}{1-\beta^2} \tag{12}$$

This implies that the equilibrium wage differential is given by:

$$w_H^*(k=1) - w_L^*(k=1) = \frac{\alpha_H - \alpha_L}{1+\beta} \tag{13}$$

[11] For expositional simplicity, our examples are "non-generic," due to the fact that all firms of a given type have exactly the same productivity. Hence in some interaction structures, all firms of a given type pay the same wage. However, since the equilibrium wage distribution is always continuous in $\boldsymbol{\alpha}$, a generic example close to the ones we discuss will have the same qualitative features, but with a completely dispersed wage distribution.

[12] Obviously, such an interaction pattern is feasible only if n is divisible by $2k$.

Consider next the case when $k = 2$ where firms are evenly spaced, and we have two H type firms, followed by two L type firms, followed by two H type firms, and so on. In this configuration each firm has one L type neighbor and one H type neighbor. Equilibrium wages, and the wage differential are now given by:

$$w_H^*(k=2) = \frac{2\alpha_H - \beta(\alpha_H - \alpha_L)}{2(1-\beta)} \tag{14}$$

$$w_L^*(k=2) = \frac{2\alpha_L + \beta(\alpha_H - \alpha_L)}{2(1-\beta)} \tag{15}$$

$$w_H^*(k=2) - w_L^*(k=2) = \alpha_H - \alpha_L \tag{16}$$

The wage differential in this case is higher than the case with alternating locations.

We now demonstrate that increased interaction between like firms yields greater dispersion in more generality. Let $F_k(w)$ denote the distribution function of wages under a k-type interaction structure, i.e., for any wage w, $F_k(w)$ is the number of firms with wage less than or equal to w. We now have the following proposition.

PROPOSITION 3. *Consider two k type interaction structures, where $k = r$ and $k = s$ where $s > r$. The equilibrium wage distribution F_r second order stochastically dominates the equilibrium wage distribution F_s.*

Proof. See Appendix.

This result is a strong one—the wage distribution when like interacts more with like is a mean preserving spread of the equilibrium wage distribution when like and unlike interact more. Since F_r stochastically dominates F_s, *any* measure of wage dispersion will be higher under s than under r.

Finally, we may also analyze the case where average productivity also differs between two labor markets. Let $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$ be two productivity n-vectors, where for $\alpha_i \in \{\alpha_H, \alpha_L\}$ for all i, and $\alpha_i' \in \{\alpha_H, \alpha_L\}$ for all i. Let $\mathbf{w}^*(\boldsymbol{\alpha})$ and $\mathbf{w}^*(\boldsymbol{\alpha}')$ be the associated equilibrium wage distributions. We now show that one may compare the wages offered by firm i in the labor market $\boldsymbol{\alpha}$ and firm j in labor market $\boldsymbol{\alpha}'$. To do this, define the m-neighbors of firm i as the m firms on either side of i who are closest to firm i. Let $\#m(i)$ denote the number of H-type firms in firm i's m-neighborhood,where m ranges from 0 to $[n/2]$—the 0-neighborhood of a firm is simply the firm itself. We say that $j \rhd i$ if for some m^*, if $m = m^*$ then $\#m(j) > \#m(i)$ and if $m < m^*$ then $\#m(j) = \#m(i)$. In other words, $j \rhd i$ if the types of the *nearest* neighbors who differ between firm i and firm j are such that there there are more H type nearest neighbors for firm j than for firm i.

PROPOSITION 4. *Suppose that $\boldsymbol{\alpha}, \boldsymbol{\alpha}'$ are two productivity n-vectors where each component takes values in $\{\alpha_H, \alpha_L\}$, and firm j belongs to $\boldsymbol{\alpha}$ and firm i to $\boldsymbol{\alpha}'$.*

i) If $j \rhd i$, then $w_j^*(\boldsymbol{\alpha}) > w_i^*(\boldsymbol{\alpha}')$.
ii) If neither $j \rhd i$ nor $i \rhd j$, then $w_j^*(\boldsymbol{\alpha}) = w_i^*(\boldsymbol{\alpha}')$.

Proof. See Appendix.

Given firm i from $\boldsymbol{\alpha}$ and firm j from $\boldsymbol{\alpha}'$, we can determine which firm offers higher wages by a lexicographic procedure. If the two firms are of different productivity, the higher productivity firm will offer the higher wage; otherwise, we compare the average producitivity i's closest pair of neighbors and j's closest neighbors, and whichever is greater will be the firm offering higher wages. More generally, if the number of high and low productivity firms in each pair of neighbors up to $m^* - 1$ places away are identical for firm i and firm j, but the pair of neighbors m^* places away from j include more high productivity firms than i then firm j's equilibrium wage is higher.

Note that high productivity firms always offers higher wages than a low productivity firms, irrespective of outside factors, provided that there are only two possible productivity levels in the labor market. However, this is not true more generally, as may be seen by considering the example when $k = n/2$. In this example, two firms of the same productivity end up offering different wages. Since the wage distribution $\mathbf{w}^*$ is continuous in the vector of productivities $\boldsymbol{\alpha}$, if one increases slightly the productivity of the firm offering the lower wage, it will still offer lower wages than the other firm. This point illustrates that in more general cases, both inside and outside factors matter for wage determination, so that one cannot make predictions based solely on one factor.

3.2. **Establishment Size.**

Given equilibrium wage rates, we can write the equilibrium labor supply to each firm as

$$L_i^* = \frac{1}{n} - \frac{2v\delta}{t} + \frac{1+2\delta}{t} w_i^* - \frac{1}{2t} w_{i+1}^* - \frac{1}{2t} w_{i-1}^*. \tag{17}$$

Substituting for equilibrium wages, this can be rewritten in matrix notation as $\mathbf{L}^* = 1/n - 2v\delta/t + RQ\boldsymbol{\alpha}$ where

$$R = \begin{bmatrix} \frac{1+2\delta}{t} & -\frac{1}{2t} & 0 & \cdots & 0 & -\frac{1}{2t} \\ -\frac{1}{2t} & \frac{1+2\delta}{t} & -\frac{1}{2t} & & 0 & 0 \\ 0 & -\frac{1}{2t} & \frac{1+2\delta}{t} & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & -\frac{1}{2t} & 0 \\ 0 & 0 & & -\frac{1}{2t} & \frac{1+2\delta}{t} & -\frac{1}{2t} \\ -\frac{1}{2t} & 0 & \cdots & 0 & -\frac{1}{2t} & \frac{1+2\delta}{t} \end{bmatrix}. \tag{18}$$

Letting $Z = RQ$, it is clear that since both R and Q are circulant, Z is also circulant.[13] Furthermore, it can be shown that the first line of Z is given by

$$\mathbf{z} = \left(\frac{q_0 - \beta q_1}{t\beta}, -\frac{q_1}{2t\beta}, -\frac{q_2}{2t\beta}, \ldots, -\frac{q_{n-1}}{2t\beta} \right). \tag{19}$$

It can be straightforwardly shown that:

PROPOSITION 5. *For* $i = 0, 1, \ldots n-1$,

$$L_i^* = \frac{1}{n} - \frac{2v\delta}{t} + \sum_{j=0}^{n-1} z_j \alpha_{i+j} \tag{20}$$

where $z_j = z_{n-j}$ *for* $j = 1, \ldots [n/2]$ *and* $z_0 > -z_1 > -z_2 > \ldots > -z_{[n/2]} > 0$.

That is, firm i's employment depends positively on its own productivity, negatively on rival productivities, the effect of rival productivities is symmetric for equally distant rivals and this effect is declining in distance.

We can now use this characterization of firm employment to examine the relative size of various establishments, restricting ourselves to markets where employers can be either high or low productivity. Begin by considering the first two examples from the previous section. The size differential of high productivity firms in comparison to low productivity firms are given by:

$$L_H^* - L_L^* = \frac{2(1+\delta)}{t}(w_H^* - w_L^*) \tag{21}$$

$$L_H^* - L_L^* = \frac{1+2\delta}{t}(w_H^* - w_L^*) \tag{22}$$

[13] A square matrix C is *circulant* if the elements of each row of C are identical to those of the previous row, but are moved one position to the right and wrapped around (Davis, 1979).

for $k = 1$ and $k = 2$. Thus for these examples, employers that offer higher wages employ more workers.

More generally, we can characterize the relative sizes of any two establishments for labor markets with two types of employers.

PROPOSITION 6. *Suppose that $\boldsymbol{\alpha}, \boldsymbol{\alpha}'$ are two productivity n-vectors where each component takes values in $\{\alpha_H, \alpha_L\}$, and firm j belongs to $\boldsymbol{\alpha}$ and firm i belongs to $\boldsymbol{\alpha}'$.*

i) If $j \rhd i$ and $m^* = 0$ (i.e., $\alpha_j = \alpha_H$ and $\alpha_i = \alpha_L$) then $L_j^*(\boldsymbol{\alpha}) > L_i^*(\boldsymbol{\alpha}')$.
ii) If $j \rhd i$ and $m^* > 0$ then $L_j^*(\boldsymbol{\alpha}) < L_i^*(\boldsymbol{\alpha}')$.
iii) If neither $j \rhd i$ nor $i \rhd j$ then $L_j^*(\boldsymbol{\alpha}) = L_i^*(\boldsymbol{\alpha}')$.

Proof. See Appendix.

One implication of this proposition is that high productivity employers are always larger than low productivity employers. From Proposition 4, we know that high productivity employers also offer higher wages. Thus, we have an "employer size–wage effect" (Brown and Medoff, 1989). However, even restricting to just two productivities, it is not always the case that high wage firms are larger. Consider the case where $k = 3$. From the proof of Proposition 3 (see Appendix) we know that the wage offered by the employer in the middle of a high productivity cluster ($w_H^2(3)$) is higher than the wage offered by its neighbors ($w_H^1(3)$). But we also know from the above proposition that the middle employer is also smaller than its neighbors.

3.3. **Profitability.** There is also evidence that firms that are more profitable tend to offer higher wages (Blanchflower et al., 1996; Dickens and Katz, 1987a; Pugel, 1980). If capital and labor are the only inputs to production, this correlation is immediate.

Suppose on the other hand that there is at least one other factor of production with fixed cost c_i for firm i. For example, in retail establishments, this factor could be the right to use a brand name (i.e., a franchise fee). Alternatively, it could be a fee paid to the owner of a scare resource (e.g., a patent holder). Suppose that this fixed cost of production is is correlated with the marginal revenue product of labor. (This correlation could arise through the bargaining over rents between the owner of the resource and the firm.) For illustrative purposes, suppose that this correlation is perfect and that $c_i = \gamma\phi_i$. In this case, profits can be rewritten as:

$$\pi_i = \phi_i(L_i - \gamma) - w_i L_i \tag{23}$$

Although it is not necessarily true that high productivity firms offer higher wages (see page 12), it will be true *on average*. In this case, higher productivity employers will typically earn higher profits *and* pay higher wages. Note that even though the correlation between c_i and ϕ_i is perfect, the correlation between π_i and w_i will be imperfect and depends on how employers are distributed in relation to one another.

In sum, by imposing some additional structure on the fixed cost of production, our model can explain the observed correlation between profitability and wages. The degree of correlation depends on the nature of interaction between employers and on the degree of correlation between productivity and fixed production costs.

3.4. **The Effect of Minimum Wages on the Distribution of Wages.** We now consider the effect of minimum wages on the distribution of wages. Empirical work on minimum wages has noted that minimum wages tend to have spillover effects on high wage firms and reduces wage dispersion. In addition, the wage distribution usually has a "spike" at the minimum wage, with dispersed wages above it. This is particularly interesting since theoretical models have difficulty in generating a wage distribution which combine a mass point and a continuous density.

If a minimum wage w^m is imposed, each firm's optimal wage is given by:

$$\tilde{w}_i = \max\left\{\left[\alpha_i + \frac{\beta}{2}(w_{i-1} + w_{i+1})\right], w^m\right\} \tag{24}$$

We can use matrix notation to define the optimal wage-setting rule for all firms as:

$$\tilde{\mathbf{w}} = (\boldsymbol{\alpha} + B\,\mathbf{w}) \vee \mathbf{w}^m \tag{25}$$

where $\mathbf{w}^m$ denotes the vector which has all entries as w^m, and the operation $\vee$ denotes the join of two vectors, i.e., the vector of component wise maxima. Although the wage setting rule is now no longer linear, it is still increasing and a contraction mapping. The equilibrium is now given by a fixed point of this mapping.

PROPOSITION 7. *Under a minimum wage, the Nash equilibrium, exists and is unique. Furthermore, it is such that*

i) if the minimum wage is strictly binding for any firm, it strictly increases the wage offered by every *firm,*

ii) if a minimum wage binds on any firm, it strictly reduces the difference between lowest wage and every other wage paid in this market, and *iii) a minimum wage robustly gives rise to a spike in the wage distribution, i.e., for any minimum wage w^m, the set of productivity vectors $\boldsymbol{\alpha}$ such that $w_i(\boldsymbol{\alpha}, w^m) = w_j(\boldsymbol{\alpha}, w^m) = w^m$ for $i \neq j$ contains an open set of positive Lebesgue measure.*

Proof. See Appendix.

We illustrate these results by re-examining our previous examples. Returning to the case where high and low productivity firms alternate in location, consider the effect of a minimum wage, starting at w_L^*, the equilibrium wage of the less productive firms. An increase in the minimum wage above w_L^* affects the wage paid by the less productive firms one-for-one. The effect on wages paid by the high type firms is given by their reaction function, as long as this wage is above the minimum. Thus although the minimum wage is not binding on the high wage firms, there is a spillover effect which is due to the strategic complementarity in wage setting. Finally, at the point w', where the reaction function intersects the $45°$ line, the minimum wage becomes binding for the high productivity firms as well and they also start paying exactly the minimum wage.

Consider the implications of this example for wage dispersion. A minimum wage reduces the range of wages paid, but to a smaller extent than if there were no interaction, since wages also rise in the high wage firms. Note that the difference between the wages paid by the two types of firms at any minimum wage is given by the vertical distance between the high productivity firm's reaction function and the $45°$ line. This declines with the minimum wage, and finally shrinks to zero at w'.

Now consider again the configuration where $k = 2$. The wage of the high productivity firms, as a function of the wage of low productivity firms, is given by:

$$w_H = \frac{2\alpha_H}{2-\beta} + \frac{\beta}{2-\beta} w_L. \tag{26}$$

Hence a minimum wage which binds only on low wage firms raises the wages of high productive firms by a factor $\beta/(2-\beta)$. This effect is positive, but less than the effect in the case of alternating firms since $\beta < 1/2$. Indeed, one can show that high wage firms here are hurt less than in the former case.

These examples show that when high wage firms interact directly with only low wage firms, there is less wage dispersion and a minimum wage reduces this dispersion quickly. When high wage firms interact with both low wage and high wage firms, there is greater wage dispersion and a minimum wage compresses this dispersion at a slower rate.

We can also see how the minimum wage robustly generates a spike in the wage distribution. Consider for example the case when $k = 2$, where all H-type firms offer the same high wage while all $L-$type firms offer the same low wage. This example is "non-generic," since all firms of a given type have exactly the same productivity. However, we can modify this example slightly, so that L type producitivities are slightly different from each other, all H type producitivities are also slightly different from each other, while there is a large difference between the producitivities of two firms of different types. By continuitiy, the corresponding equilibrium wage distribution would be close to the one we have analyzed, but with slight wage dispersion within the class of high types and within the class of low types. A minimum wage would quickly bind upon all the low-type firms, ensuring that they all pay the same wage, irrespective of the slight differences in productivity. On the other hand, wage dispersion would continue within the class of high type firms since the minimum wage will not bind upon them. Hence a mixed wage distribution is very easily generated by our model.

Finally, consider the effect of a minimum wage on firm profits. Although it is widely thought that a minimum wage affects mainly low wage firms, on whom the minimum binds, this intuition is incorrect. Consider any firm i and the effect of a small minimum wage, w^m, which just binds on the lowest wage firms. The envelope theorem tells us that the effect of a rise in a firm's own wage, from w_i^* to $\hat{w}_i(w^m)$, has a zero effect upon the firm's profits. However, the rise in competitors' wages, (i.e., the wages paid by firms $i-1$ and $i+1$) reduces the labor supply to firm i, thus reducing its profits.

$$\left.\frac{d\pi_i}{dw^m}\right|_{w^m=w_j^*} = \frac{\partial \pi_i}{\partial w_{i-1}}\frac{\partial w_{i-1}}{\partial w^m} + \frac{\partial \pi_i}{\partial w_{i+1}}\frac{\partial w_{i+1}}{\partial w^m} \tag{27}$$

where w_j^* is the lowest wage paid in the market prior to the imposition of the minimum wage. Since the minimum wage has the largest impact upon low wage firms, the above expression indicates that the adverse effects on profitability are likely to be suffered mostly by those firms who compete with low wage firms. To investigate this further, assume that only one firm, firm j pays the lowest wage (we know from Proposition 1 that this is generically true). The effects on the wage distribution of a minimum wage which only binds upon this firm can duplicated by replacing productivity profile $\boldsymbol{\alpha}$ with $\boldsymbol{\alpha}'$

where $\alpha'_i = \alpha_i$ for all $i \neq j$ and $\alpha'_j > \alpha_j$. Hence the effects on profits are given by

$$\begin{aligned} \left.\frac{d\pi_0}{d\alpha'_j}\right|_{\alpha'_j=\alpha_j} &= \left.\frac{\partial \pi_0}{\partial w_1}\frac{\partial w_1}{\partial \alpha'_j}\right|_{\alpha'_j=\alpha_j} + \left.\frac{\partial \pi_0}{\partial w_{n-1}}\frac{\partial w_{n-1}}{\partial \alpha'_j}\right|_{\alpha'_j=\alpha_j} \\ &= -(\phi_0 - w_0^*)\frac{(q_{j-1}+q_{j+1})/2}{t} \end{aligned} \tag{28}$$

Firm 0 suffers a negative effect on its profits because the rise in the wages of its competitors reduces its labor supply. Firstly, the magnitude of this effect depends on it's profitability, $\phi_0 - w_0^*$. The more profitable it is prior to the minimum wage, the more it is hurt. Second, it depends on the degree to which the minimum wage affects its competitor's wages, $(q_{j-1} + q_{j+1})/2$. That is, the more directly it competes with a minimum wage firm, the more it is hurt. Interestingly, since the minimum wage firm does not compete with itself, it can be less injured than its immediate neighbors: it is typically less profitable and its direct competitors raise their wages by less than their direct competitors.

4. Related Literature

Efficiency wage theory has been offered as one explanation for wage dispersion. Ramaswamy and Rowthorn (1991) consider an efficiency wage model where firms have heterogeneous production functions, and assume that effort in each firm is a function of the wage—the micro-foundations behind this effort decision are not specified. Each firm sets the wage to satisfy a generalized Solow condition and this gives rise to wage dispersion. Since effort does not depend upon outside wages, there are no spillover effects, and hence minimum wages would not affect high wage firms. For the same reason, this model can also accommodate a spike in the wage distribution caused by a minimum wage.

Albrecht and Vroman (1998) consider an efficiency wage model with homogeneous firms where workers differ in their disutility of effort, so that there is adverse selection in addition to moral hazard. For any given wage, the set of employees of the firm is partitioned into shirkers (those with a relatively high disutility of effort) and non-shirkers. When there is a continuous wage distribution, firms face a smooth trade-off, where a higher wage reduces the set of shirkers, and increases aggregate effort. When the wage distribution has a mass point, however, a firm at the mass point has an incentive to offer a slightly higher wage because by doing so it can discontinuously increase the proportion of non-shirkers amongst its new hires. Thus the equilibrium in this model must not only involve wage dispersion but the distribution of wages must be atomless. As a result, there cannot be a spike in the wage distribution.[14]

An alternative approach is the literature on job search. Workers must search in order to know about wage offers, and this gives employers market power. Burdett and Mortensen (1998) analyze wage dispersion in a model with a fixed number of firms where workers search both when employed and unemployed. The equilibrium wage distribution is atomless and lies below the marginal product of labor with larger firms offering higher wages. Firms are indifferent between all wages in its support, since they attract more workers by offering higher wages. A minimum wage shifts the distribution upward so that there is a spillover effect. However, like Albrecht and Vroman (1998), it must also remain atomless, because otherwise a firm would be able to discontinuously increase its labor supply by a small increase in the wage. Hence this model does not explain the observed spike in the wage-distribution induced by minimum wages.

Furthermore, the papers by Albrecht and Vroman (1998) and Burdett and Mortensen (1998) have very strong empirical predictions regarding the shape of the wage distribution. In particular, the density function over equilibrium wages in Albrecht and Vroman (1998) must be monotonically decreasing in the wage rate. In contrast, Burdett and Mortensen (1998) predicts that the density function should be monotonically increasing in the wage rate. That is, for identically able workers, the relative frequency of a wage offer is a monotonic function of the wage—either case seems implausible as a general rule. Burdett and Mortensen (1998) provide an extension which—like the current paper—requires employers to be heterogeneous in terms of their productivity. Once employer productivity differences are allowed for, non-monotonic equilibrium wage distributions can emerge. That is, in order to produce an interesting wage distribution, they must also allow for employer heterogeneity. However, as before, these wage distributions must remain atomless and therefore

[14]The literature on employee turnover Salop (1979a) is also related. This is motivated by the notion that workers are unsure of employer characteristics prior to employment and only learn about them gradually, however, turnover is determined exogenously and lacks microfoundations (i.e., worker quit decisions are left unmodeled).

even with employer heterogeneity, Burdett and Mortensen (1998) is unable to explain the existence of a spike at the minimum wage.

The model presented here has focused on wage dispersion, given firm heterogeneity. In related work (Bhaskar and To, 1999), we used a symmetric model with identical firms in order to examine the employment effects of a minimum wage. The basic result is that employment can increase with a minimum wage if the heterogeneity in preferences and the fixed costs of production are "significant." That is, Card and Krueger's (1994) "paradoxical" results may not be so paradoxical if these labor market distortions are relatively large. In order to consider minimum wages in a convincing setting, we allowed for free entry and exit but as a result, for tractability, were forced to impose employer symmetry. As we've shown, employer symmetry implies uniform wage offers and therefore no wage dispersion.

5. Concluding Remarks

The predictions of our model closely match the stylized facts of wage dispersion and in some respects, our approach performs better than the existing literature. Search and efficiency wages are no doubt also important to our understanding of the functioning of labor markets. However, these models, which are the most popular explanations for wage dispersion, are inconsistent with the existence of a spike in the wage distribution and are thus not completely satisfactory as explanations of wage dispersion. More importantly, oligopsony is extremely tractable, lending itself to both theoretical and empirical applications. The specific model used is, admittedly, highly stylized because of our desire to construct a model of oligopsony based on primitives. However, the basic point being made is quite general—once workers have heterogeneous preferences, firm level labor supply curves are upward sloping and an employer offering a low wage can still have positive employment. In sum, oligopsony appears to perform at least as well as search and efficiency wage models with the added benefit of tractability.

Appendix

The proofs of several of our results rely on the fact that the optimal wage setting functions, (8) and (25) are both contraction mappings, and are also increasing. We demonstrate these below:

Proof that (8) is a contraction mapping. Given $\mathbf{y}, \mathbf{y}' \in \mathbb{R}^n$, let $d(\mathbf{y}, \mathbf{y}') = \max_i |y_i - y_i'|$. Let $\mathbf{w}$, $\mathbf{w}'$ be two wage vectors, and let $\mathbf{f}$, $\mathbf{f}'$ be the associated optimal wages given by (8). For any i, we have

$$\begin{aligned}|f_i - f_i'| &= \frac{\beta}{2}\big|(w_{i-1} - w_{i-1}') + (w_{i+1} - w_{i+1}')\big| \\ &\leq \frac{\beta}{2}[2d(\mathbf{w}, \mathbf{w}')]\end{aligned} \tag{29}$$

Since $d(\mathbf{f}, \mathbf{f}') \leq \beta d(\mathbf{w}, \mathbf{w}')$ and $\beta < 1/2$, (8) is a contraction mapping. ■

Proof that (25) is a contraction mapping. Let $\mathbf{g}$, $\mathbf{g}'$ be the associated optimal wages given by (25) and some minimum wage w^m. For any i, we establish that $|g_i - g_i'| \leq |f_i - f_i'|$, which suffices to prove the required result. If $f_i > w^m$ and $f_i' > w^m, g_i - g_i' = f_i - f_i'$. If $f_i \leq w^m$ and $f_i' \leq w^m, g_i - g_i' = 0$. If $f_i > w^m$ and $f_i' \leq w^m$, $0 < g_i - g_i' = f_i - w^m \leq f_i - f_i'$. Hence in any case $|g_i - g_i'| \leq |f_i - f_i'|$. ■

The contraction mapping property has several implications. First, equilibrium exists and is unique, in each case. Second, from *any* initial vector of wages, $\mathbf{w}(1)$, the dynamic process defined by iterated application of the mapping results in convergence to the equilibrium.

We call a function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ *increasing* if $\mathbf{y} \geq \mathbf{y}' \Rightarrow \mathbf{f}(\mathbf{y}) \geq \mathbf{f}(\mathbf{y}')$. It is easy to verify that (8) and (25) are both increasing functions.

Proof of Proposition 1. Existence and uniqueness of Nash equilibrium is implied by the existence of the inverse matrix, $Q = (I - B)^{-1}$. We solve for this inverse as follows. Employer i's equilibrium wage is given by $w_i^* = \sum_{j=0}^{n-1} q_{ij}\alpha_j$. Since $I - B$ is a symmetric circulant matrix, Q is also a symmetric circulant matrix. Circulant matrices can be defined by their first row so let $\mathbf{q} = (q_0, q_1, \ldots q_{n-1}) = (q_{0,0}, q_{0,1}, \ldots, q_{0,n-1})$. Noting that $Q(I - B) = I$, it is easy to see that $\mathbf{q}$ must solve:

$$q_0 - \frac{\beta}{2}q_1 - \frac{\beta}{2}q_{n-1} = 1 \tag{30}$$

$$-\frac{\beta}{2}q_j + q_{j+1} - \frac{\beta}{2}q_{j+2} = 0 \tag{31}$$

for $j = 0, 1, \ldots n - 3$ and

$$-\frac{\beta}{2}q_0 - \frac{\beta}{2}q_{n-2} + q_{n-1} = 0 \tag{32}$$

Notice that (31) is a second order linear difference equation with characteristic roots:

$$\lambda = \frac{1}{\beta} - \sqrt{\left(\frac{1}{\beta}\right)^2 - 1} \tag{33}$$

$$\mu = \frac{1}{\beta} + \sqrt{\left(\frac{1}{\beta}\right)^2 - 1} \tag{34}$$

and since $\beta < 1/2$, it follows that $0 < \lambda < 1 < \mu$. The general solution to (31) is therefore

$$q_j = A\lambda^j + B\mu^j \tag{35}$$

for arbitrary constants A and B. Substituting this into (30) and (32) results in a system of two equations with two unknowns, A and B. Solving yields:

$$A = \frac{1}{(1-\lambda^n)\sqrt{1-\beta^2}} \tag{36}$$

$$B = \frac{1}{(\mu^n - 1)\sqrt{1-\beta^2}}. \tag{37}$$

These are both positive and therefore $q_t > 0$ for all $j = 0, \ldots n-1$.

Since Q is symmetric and circulant, it must be the case that $q_j = q_{n-j}$ for $j = 1, \ldots [n/2]$. Furthermore, since $\lambda < 1$ and $\mu > 1$, if q_j is non-monotonic in j, it must first be declining and then be rising. But because Q is symmetric, it must be the case that $q_0 > q_1 > \ldots > q_{[n/2]}$.

Finally, in order to ensure that the equilibrium is well behaved, we need to bound the productivities, given parameters δ, v, n and t. In equilibrium, we must have that for all i and $j = i-1, i+1$, i) $|w_i - w_j| \leq t/n$ (i.e., all low reservation wage workers work), ii) $x_i^v \leq 1 - x_j^v$ (i.e., some high reservation wage workers are unemployed), iii) $x_i^v \geq 0$ and iv) Finally, we also need to ensure that the wage setting rule, which satisfies the first order condition for profit maximization, is also globally optimal. That is, we need to ensure that no firm will choose to offer a wage so high that its neighbor employs no workers.

i) Take two competitors 0 and 1. The maximum value that $w_0^* - w_1^*$ can take is when, the α_j assume the minimum value, α_*, for $j = 1, \ldots [n/2]$ and the α_j assume the maximum value, α^*, for $j = [n/2]+1, \ldots n$. This implies the bound

$$(\alpha^* - \alpha_*) \leq \frac{1}{q_0 - q_{[n/2]}} \frac{t}{n}$$

ii) The maximum possible equilibrium wage occurs when all employers have the maximum productivity α^*. A straightforward comparison shows that the inequality, $x_i^v \leq 1/n - x_j^v$, will be satisfied whenever

$$\alpha^* \leq \frac{t/2n + v}{\sum_{j=0}^{n-1} q_j}$$

iii) The minimum possible equilibrium wage occurs when all employers have the minimum productivity α_*. A straightforward comparison shows that the inequality, $x_i^v \geq 0$, will be satisfied whenever

$$\alpha_* \geq \frac{v}{\sum_{j=0}^{n-1} q_j}$$

iv) A sufficient condition for firm a firm to be unwilling to choose a wage so high as to capture its neighbors' markets is that the net revenue product should be no greater than the wage which would be sufficient to capture the neighboring labor market—that is for firm 1, $\phi_1 \leq \bar{w}_1^* + t/n$.[15] This is satisfied whenever

$$\left(1 - \frac{q_1}{2}\right)\alpha_1 \leq \frac{1}{2}\left[q_0\alpha_0 + \sum_{j=2}^{n-1} q_j\alpha_j\right] + \frac{(1+3\delta)t/n + 2\delta v}{2(1+\delta)}$$

Evaluating this when $\alpha_1 = \alpha^*$ and $\alpha_j = \alpha_*$ for $j \neq 1$, this can be rewritten as

$$\left(1 - \frac{q_1}{2}\right)\alpha^* \leq \frac{1}{2}\left(\frac{1}{\sqrt{1-\beta^2}} - q_1\right)\alpha_* + \frac{(1+3\delta)t/n + 2\delta v}{2(1+\delta)}$$

Although each α_i is a function of the parameters, it is also linear in ϕ_i so that given these parameters, α_i can take any positive value for appropriately chosen ϕ_i. i) specifies a maximum range over which the α's can span. The bounds given by iii) is always lower than that given by ii). The bounds given in ii) and iv) will be greater than α_* provided that t, δ and v are sufficiently large. ■

[15] If this condition is satisfied, then firm 1 will also not choose to capture markets 2, 3... $[n/2]$ places away because wage required to do so will be even greater and so will also exceed the net revenue product.

Proof of Proposition 2. i) Since $\alpha_j \geq \alpha'_j$ for all j and $\alpha_j > \alpha'_j$ for some j,

$$\begin{aligned} w_i^* &= \sum_{j=0}^{n-1} q_j \alpha_{i+j} \\ &> \sum_{j=0}^{n-1} q_j \alpha'_{i+j} = w_i^{*\prime} \end{aligned} \tag{38}$$

for every i. Therefore, $\mathbf{w}^* \gg \mathbf{w}^{*\prime}$.

ii) By assumption, $\sum_{i=0}^{n-1} \alpha_i = \sum_{i=0}^{n-1} \alpha'_i$. Writing out the sum of the wages,

$$\begin{aligned} \sum_{i=0}^{n-1} w_i^* &= \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} q_j \alpha_{i+j} = \sum_{j=0}^{n-1} q_j \sum_{i=0}^{n-1} \alpha_{i+j} \\ &= \sum_{j=0}^{n-1} q_j \sum_{i=0}^{n-1} \alpha'_{i+j} = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} q_j \alpha'_{i+j} = \sum_{i=0}^{n-1} w_i^{*\prime} \end{aligned} \tag{39}$$

Therefore, $M(\mathbf{w}^*) = M(\mathbf{w}^{*\prime})$.

iii) Suppose that $w_i^*(\boldsymbol{\alpha}) = w_j^*(\boldsymbol{\alpha})$ for $i \neq j$. Hence

$$\sum_{h=0}^{n-1} q_h[\alpha_{i+h} - \alpha_{j+h}] = 0 \tag{40}$$

Hence if $\boldsymbol{\alpha}$ satisfies equation (40), it must lie in a hyperplane of lower dimension, and hence the set of $\boldsymbol{\alpha}$ satisfying this equation is of Lebesgue measure zero in $\mathbb{R}^n$.

■

To prove Propositions 3–6 we first prove the following Lemma. We then use the result of Lemma 1 to prove Proposition 4 which is in turn used to prove Proposition 3. Finally, we prove Proposition 6. Prior to giving the statement of the Lemma and subsequent proof, we define the following, additional notation.

Let S_h be defined as follows:

$$S_h = \begin{cases} 2\sum_{j=h}^{[n/2]} q_j & \text{if } n \text{ is odd} \\ 2\sum_{j=h}^{n/2-1} q_j + q_{n/2} & \text{if } n \text{ is even} \end{cases}$$

I.e., S_h is the sum of the weights (q_j) associated with *all* the firm's neighbors who are h places away or further. The following lemma plays an important part in the proofs of Propositions 3 and 4:

LEMMA 1. $S_{h+1} < q_h$ *for all* $h \in \{0, 1, \ldots, [n/2] - 1\}$.

Proof. We show first that for any $j \leq [n/2]-2, q_t > 3q_{j+1}$ and if $j = [n/2]-1, q_t > 2q_{j+1}$. Consider the basic difference equation for q_j

$$q_j = \frac{2}{\beta}q_{j+1} - q_{j+2} \tag{41}$$

Since $\beta < 1/2$ and $q_{j+2} < q_{j+1}$ if $j+2 \leq [n/2]$, it follows that

$$q_j > 4q_{j+1} - q_{j+2} > 3q_{j+1} \tag{42}$$

Which proves the required result. For the case when $j = [n/2]-1$, we have that $q_{j+2} = q_j$, which implies $q_j > 2q_{j+1}$. Using this result, we have:[16]

$$S_h < 2q_h \left[\sum_{j=0}^{[\frac{n}{2}]-h-1} \left(\frac{1}{3}\right)^t \right] + 2q_h \left(\frac{1}{3}\right)^{[\frac{n}{2}]-h-1} \times \frac{1}{2} = 3q_h < q_{h-1}$$

which proves the lemma. ■

Proof of Proposition 4. i) Let $j \rhd i$, and consider the extreme case when $\#m^*(j) = \#m^*(i)+1$ and all firm j's neighbors who are m^*+1 or more places away are of low productivity whereas all of firm i's neighbors who are m^*+1 or more places away are of high productivity. In every instance, the difference between w_j^* and w_i^* is no less than this extreme case. Letting $\triangle\alpha = (\alpha_H - \alpha_L)/2$, this difference is:

$$w_j^*(\boldsymbol{\alpha}) - w_i^*(\boldsymbol{\alpha}') \geq 2\triangle\alpha\left(q_{m^*} - S_{m^*+1}\right) > 0$$

which establishes the first part of the proposition.

ii) If neither $j \rhd i$ nor $i \rhd j$ apply, for $l = 1, \dots [n/2]$, each pair of neighbors l places from j have the same productivities as those l places from i and hence their wages are identical.

■

Proof of Proposition 3. For any k, let $w_H^1(k)$ denote the lowest wage paid by an H-type firm. This wage will be paid by a firm which is at the edge of its cluster, i.e., it is located adjacent to an L-type. Similarly, let $w_H^l(k)$ be the wage paid by an H type whose nearest L-type neighbor is l places away. We shall say that this firm's location is of type l. Clearly, l ranges from 1 to $[(k+1)/2]$, so that there are $[(k+1)/2]$ wages paid by H type firms in a k distribution. Let $w_L^1(k)$ denote the highest wage paid by an L type firm, i.e., this is the wage paid by a firm which is adjacent to an H type. Also, let $w_L^l(k)$ be

[16] The expression below uses the definition of S_h when n is odd—this is sufficient, since this expression is always greater than the definition when n is even.

the wage paid by a L type whose nearest H-type neighbor is l places away. The following statements are easily verified:

a) If firm i is of type L and j is of type $H, j \rhd i$. Hence $w_L^l(k) < w_H^{l'}(k)$ for any l, l'.
b) If both firms are of type H, with firm i's location of type l and firm j's location of type $l' > l$, then $j \rhd i$. Hence $w_H^l(k) < w_H^{l'}(k)$ if $l' > l$.
c) If both firms are of type L, with firm i's location of type l and firm j's location of type $l' > l$, then $i \rhd j$. Hence $w_H^l(k) > w_H^{l'}(k)$ if $l' > l$.

Hence we have established that:

$$w_L^{[k/2]}(k) < w_L^{[k/2]-1}(k) < \ldots < w_L^2(k) < w_L^1(k) < w_H^1(k) < w_H^2(k) < \ldots < w_H^{[k/2]-1} < w_H^{[k/2]}$$

Furthermore, it is straightforward to see that for any k, the distribution of offered wages is symmetric, so that $w_H^l(k) - \bar{w} = \bar{w} - w_L^l(k)$ for any l, where $\bar{w} = (\alpha_H + \alpha_L)/2 \sum_{j=0}^{n-1} q_j$ is the mean wage, which does not depend upon k.

We now compare wages across interaction structures s and r where $s > r$. Note that:

d) $w_H^l(s) > w_H^l(r)$ for any $l \in \{1, 2, \ldots, [r/2]\}$. This follows from the fact that if we consider a pair of H-type firms, where j is of location type l in interaction structure s, while i is of location type l in interaction structure r, then $j \rhd i$.
e) If $[r/2] < l \leq [s/2]$, then $w_H^l(s) > w_H^{[r/2]}(r)$ where the latter is the largest wage paid in interaction structure r. This follows from the fact that if we consider a pair of H-type firms, where j is of location type l in interaction structure s, while i is of location type $[r/2]$ in interaction structure r, where $l > [r/2]$, then $j \rhd i$.

The symmetry of the wage distribution for any k implies that analogous comparative statements can be made about wages offered by low type firms in the two interaction structures. I.e., we have:

f) $w_L^l(s) < w_L^l(r)$ for any $l \in \{1, 2, \ldots, [r/2]\}$.
g) If $[r/2] < l \leq [s/2]$, then $w_L^l(s) < w_L^{[r/2]}(r)$.

We are now ready to establish that the wage distribution under interaction structure s second order stochastically dominates the wage distribution under interaction structure r. To do this, we construct a one-to-one mapping from the set of H type firms under s and the set of H type firms under r such that if j is the image of i, then the wage offered by j is strictly greater than the wage offered by i.

Begin by taking a cluster of H type firms under r (this cluster is of size r) and a cluster of H type firms under s (this cluster is of size s). Since we have established that $w_H^l(s) > w_H^l(r)$ for every $l \leq [r/2]$, matching firms with the same value of l in these clusters suffices. Since the two clusters are of unequal sizes, this leaves us with the firms in interaction structure s who are in the middle of the cluster, who are unmatched. Proceed by undertaking the same exercise with another pair of clusters, until all the clusters from interaction structure s have been used up. However, we have established that if $l > [r/2]$, $w_H^l(s)$ is greater than any wage offered in interaction structure r. Hence we may match the remaining clusters in r with these high wage firms in s in any way while meeting the requirement.

A symmetric argument establishes that we can construct a one-to-one mapping from the set of L type firms under s and the set of L type firms under r such that if j is the image of i, then the wage offered by j is strictly smaller than the wage offered by i. This completes the proof of the proposition. ∎

Proof of Proposition 6. i) To prove the second part, we first need to show that $z_0 > -3z_1$. Writing out z_0 we see that,

$$\frac{q_0 - \beta q_1}{t\beta} > \frac{6-2\beta}{2t\beta}q_1 > 3\frac{q_1}{2t\beta} = -3z_1.$$

Using the argument from Lemma 1, it is now easy to show that for n odd, $-2\sum_{l=1}^{[n/2]} z_l < z_0$ and n even, $-2\sum_{l=1}^{n/2-1} z_l + z_{n/2} < z_0$. The remainder of the proof follows from arguments similar to that above and from Proposition 3.

ii) For $m^* > 0$, consider the extreme case where $\#m^*(j) = \#m^*(i) + 1$ and all of firm j's neighbors who are $m^* + 1$ or more places away are of low productivity whereas all of firm i's neighbors who are $m^* + 1$ or more places away are of high productivity, it follows from Lemma 1 that

$$L_i^*(\boldsymbol{\alpha}) - L_j^*(\boldsymbol{\alpha}') \geq \frac{\Delta\alpha}{t\beta}(q_m^* - S_{m^*+1}) > 0,$$

establishing the first part of the proposition.

iii) Identical to part ii) of the proof of Proposition 4.

∎

Proof of Proposition 7. i) Suppose that we are at the pre-minimum wage equilibrium, $\mathbf{w}(0)$. Consider the following tâtonnement adjustment process (in fictional time) to the new equilibrium. In period 1, all firms whose wage is below the new minimum raise their wage to w^m. In any period $t > 1$, all firms choose wages as an optimal response to $t-1$ period wages, i.e., the wage dynamic is given by (25). Note first that this process is monotone, i.e., $\mathbf{w}(t+1) \geq \mathbf{w}(t)$. This follows by induction: since (25) is an increasing function, $\mathbf{w}(t) \geq \mathbf{w}(t-1) \Rightarrow \mathbf{w}(t+1) \geq \mathbf{w}(t)$. Further, we have that $\mathbf{w}(1) \geq \mathbf{w}(0)$. Since (25) is a contraction mapping, this dynamic process converges to the new equilibrium wage distribution. Now, if at least one firm has raised its wage at $t = 1$, this strictly increases the wage offered by its immediate neighbors at $t = 2$, its neighbors' neighbors at $t = 3$, and by induction, every single firm in this economy eventually. Hence in the new equilibrium, the wage offered by every firm is strictly larger.

ii) Let w^- denote the lowest wage in the pre-minimum wage distribution, let Z be the set of firms paying this lowest wage, and let $\triangle^- = w^m - w^-$. Consider now the same tâtonnement process, with a different initial condition. Suppose that at $t = 1$, the wage of every firm is raised by $\triangle^-$ relative to the pre-minimum wage equilibrium. In every period $t > 1$, each firm chooses a best response to the wage distribution at $t-1$. At $t = 2$, if firm i belongs to the complement of Z, then it will reduce its wage. To see this, note that its optimal wage has risen relative to the pre-minimum situation by $\max\{\beta\triangle^-, w^m - w^i\}$ which is strictly less than $\triangle^-$. Hence every firm in the complement of Z strictly reduces its wage at $t = 2$, while every firm in Z leaves its wage unchanged (since $\beta\triangle^- < \triangle^-$). Hence the wage sequence from $t = 2$ onwards is monotonically declining and converges to the new equilibrium. Since every firm in Z raises its wage in the new equilibrium by $\triangle^-$ relative to the pre-minimum wage equilibrium, whereas every firm in the complement of Z raises its wage by a strictly smaller amount, the difference between the lowest wage in the market and every other wage is reduced.

iii) Consider a pair $\boldsymbol{\alpha}$, w^m where the minimum wage strictly binds on two firms, i and j (one can always find such a pair by choosing w^m appropriately, and by choosing α_i and α_j sufficiently small). Since the equilibrium wage distribution and the optimal wage setting rule in the absence of a minimum, (8), is continuous in $\boldsymbol{\alpha}$, the minimum wage will continue to bind at any productivity vector $\boldsymbol{\alpha}'$ which is sufficiently close to $\boldsymbol{\alpha}$.

■

REFERENCES

Abowd, J. M. and T. Lemieux (1993), "The Effects of Product Market Competition on Collective Bargaining Agreements: The Case of Foreign Competition in Canada," *Quarterly Journal of Economics*, 108(4), 983–1014.

Albrecht, J. W. and S. B. Vroman (1998), "Nash Equilibrium Efficiency Wage Distributions," *International Economic Review*, 39(1), 183–204.

Bhaskar, V. and T. To (1999), "Minimum Wages for Ronald McDonald Monopsonies: A Theory of Monopsonistic Competition," *Economic Journal*, 109, 190–203.

Blackburn, M. and D. Neumark (1992), "Unobserved Ability, Efficiency Wages, and Interindustry Wage Differentials," *Quarterly Journal of Economics*, 107, 1421–1436.

Blanchflower, D. G., A. J. Oswald and P. Sanfey (1996), "Wages, Profits and Rent-Sharing," *Quarterly Journal of Economics*, 111, 227–251.

Boal, W. M. and M. R. Ransom (1997), "Monopsony in the Labor Market," *Journal of Economic Literature*, 35, 86–112.

Brown, C. and J. Medoff (1989), "The Employer Size-Wage Effect," *Journal of Political Economy*, 97, 1027–1059.

Bulow, J. I. and L. H. Summers (1986), "A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynsian Unemployment," *Journal of Labor Economics*, 4, 376–414.

Burdett, K. and D. T. Mortensen (1998), "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39(2), 257–273.

Card, D. and A. B. Krueger (1994), "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review*, 84, 772–793.

Card, D. and A. B. Krueger (1995), *Myth and Measurement: The New Economics of the Minimum Wage*, Princeton University Press, Princeton, New Jersey.

Chen, Z. and R. Lent (1992), "Supply Analysis in an Oligopsony Model," *American Journal of Agricultural Economics*, 74(4), 973–979.

Davis, P. J. (1979), *Circulant Matrices*, John Wiley and Sons, New York.

Deltas, G. (1998), "Can a Minimum Wage Increase Employment and Reduce Prices? An Analysis Based on Endogenous, Observable Effort," Working paper.

Dickens, W. and L. Katz (1987a), "Inter-Industry Wage Differences and Theories of Wage Determination," NBER Working Paper No. 2271.

Dickens, W. T. and L. F. Katz (1987b), "Inter-Industry Wage Differences and Industry Characteristics," in K. Lang and J. Leonards, eds., "Unemployment and the Structure of the Labor Market," Basil Blackwell, London, chap. 3, pp. 48–90.

Dolado, J. J., F. Felgueroso and J. F. Jimeno (1997), "The Effects of Minimum Bargained Wages on Earnings: Evidence from Spain," *European Economic Review*, 41(3–5), 713–721.

Dunlop, J. T. (1957), "The Task of Contemporary Wage Theory," in G. W. Taylor and F. C. Pierson, eds., "New Concepts in Wage Determination," McGraw Hill, New York, pp. 117–139.

Gibbons, R. and L. Katz (1992), "Does Unmeasured Ability Explain Inter-Industry Wage Differentials?" *Review of Economic Studies*, 59, 515–535.

Groshen, E. (1991), "Sources of Intra-Industry Wage Dispersion: How Much Do Employers Matter?" *Quarterly Journal of Economics*, 106, 869–884.

Grossman, J. B. (1983), "The Impact of the Minimum Wage on Other Wages," *Journal of Human Resources*, 18, 359–378.

Kaas, L. and P. Madden (1999), "Equilibrium Involuntary Unemployment with Wage and Price Setting Firms," Mimeo.

Katz, L. F. (1986), "Efficency Wage Theories: A Partial Evaluation," in S. Fischer, ed., "NBER Macroeconomics Annual 1986," MIT Press, Cambridge, MA, pp. 235–276.

Krueger, A. and L. Summers (1987), "Reflections on the Inter-Industry Wage Structure," in K. Lang and J. Leonard, eds., "Unemployment and the Structure of the Labor Market," Basil Blackwell, London.

Krueger, A. B. and L. H. Summers (1988), "Efficiency Wages and the Inter-Industry Wage Structure," *Econometrica*, 56, 259–293.

McCue, K. and W. R. Reed (1996), "New Empirical Evidence on Worker Willingness to Pay for Job Attributes," *Southern Economic Journal*, 62(3), 647–653.

Murphy, K. and R. Topel (1987), "Unemployment, Risk, and Earnings: Testing for Equalizing Wage Differences in the Labor Market," in K. Lang and J. Leonard, eds., "Unemployment and the Structure of the Labor Market," Basil Blackwell, London.

Naylor, R. (1996), "Discrimination as Collusion in Imperfectly Competitive Labour Markets," *Labour*, 10(2), 447–455.

Nickell, S. and S. Wadhwani (1990), "Insider Forces and Wage Determination," *Economic Journal*, 100, 496–509.

Pugel, T. A. (1980), "Profitability, Concentration and the Interindustry Variation in Wages," *Review of Economics and Statistics*, 62, 248–253.

Ramaswamy, R. and R. E. Rowthorn (1991), "Efficiency Wages and Wage Dispersion," *Economica*, 58, 501–514.

Salop, S. (1979a), "A Model of the Natural Rate of Unemployment," *American Economic Review*, 69, 117–125.

Salop, S. (1979b), "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10, 141–156.

Slichter, S. (1950), "Notes on the Structure of Wages," *Review of Economics and Statistics*, 32, 80–91.

Stiglitz, J. E. (1985), "Equilibrium Wage Distributions," *Economic Journal*, 95, 595–618.

Sullivan, D. (1989), "Monopsony Power in the Market for Nurses," *Journal of Law and Economics*, 32, S135–178.

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