When are Wage Differentials Compensating?

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November 2018

Abstract

The compensating wage differential literature has a long history with mixed results. In order to understand circumstances under which hedonic wage estimates can be expected to be unbiased, we evaluate compensating wage differentials under a variety of theories of labor markets. In general, in models where there is equilibrium dispersion in total job values, wage differentials are compensating. In perfectly competitive labor markets and search markets without on-the-job search, there is no equilibrium job dispersion and wage differentials are not compensating. In oligopsonistic labor markets and search markets with on-the-job search, there is equilibrium job dispersion and job differentials are not compensating.

1 Introduction

The compensating wage differential literature has a long history with mixed results. For example, ... and ... find significant estimates for the willingness-to-pay with the anticipated sign. In contrast, ... and ... find willingness-to-pay estimates that are either insignificant or have signs contrary to received wisdom. In order to understand circumstances under which hedonic wage estimates can be expected to be unbiased, we evaluate compensating wage differentials under a variety of theories of labor markets.

We begin by laying out a framework with a disamenity to examine wage differentials and whether they are "compensating." Using this framework, we discuss in turn: Rosen's (1986) clean and dirty job model, spatial oligopsony models à la Bhaskar *et al.* (2002) and Bhaskar and To (2003) and a labor analogue of the Diamond (1971) search model.

Whether wage differentials are compensating hinges on the notion of "job dispersion." By job dispersion, we mean dispersion in total job values. In the ideal case where workers have identical preferences over wages and the disamenity, job dispersion implies that each worker receives different total utility for their wage/disamenity combination. Under perfect competition or a Diamond labor search model with clean and dirty jobs, there is no job dispersion in equilibrium. But under oligopsony or a Burdett and Mortensen (1998) style labor search model (i.e., Hwang *et al.*, 1998), workers with identical preferences receive different total utilities (jobs are dispersed) and wage differences are only imperfectly informative about their preferences over the disamenity.

In occupations that are competitive or where there are no opportunities for on-the-job search, wage differentials are revealing and hedonic wage regressions yield unbiased estimates of worker willingness-to-pay for job characteristics. But in occupations dominated by oligopsonistic employers or where on-the-job search is easy, hedonic wage regressions yield misleading willingness-to-pay estimates.

2 Compensating Differentials

To analyze wage differentials and amenities under a variety of labor market theories we lay out a common framework.

Workers get utility from wages, w, and get disutility from a disamenity,

ξ. Utility is given by:

$$U_i = U(w, \xi) \tag{1}$$

where U is increasing in w and decreasing in ξ.

We say that wages, w^0 and w^1, are *compensating* for amenity values, ξ^0 and ξ^1, if and only if $U(w^0, \xi^0) = U(w^1, \xi^1)$. In other words, the wage differential, $w^1 - w^0$, compensates workers for the amenity differential, $\xi^1 - \xi^0$.

For w^0 and w^1 that are compensating for ξ^0 and ξ^1, we can write the willingness-to-pay (WTP) for $\xi^1 - \xi^0$ as

$$WTP = \frac{w^1 - w^0}{\xi^1 - \xi^0}. \tag{2}$$

Given ξ^0 and ξ^1, WTP is unique only if U is quasilinear in w and WTP is constant and independent of ξ^0 and ξ^1 only if U is linear.

2.1 Perfect Competition

Consider Rosen's (1986) model of "clean" and "dirty" jobs. Let $\xi = 0$ for clean jobs and $\xi = 1$ for dirty jobs. Since dirty jobs are less desirable, $U(w, 0) > U(w, 1)$ for any w.

Suppose that w^{0*} and w^{1*} are the equilibrium wages for clean and dirty jobs. Provided that workers are employed in both clean and dirty jobs, perfect competition ensures that workers are indifferent between clean and dirty jobs so that $U(w^{0*}, 0) = U(w^{1*}, 1)$ where $w^{1*} > w^{0*}$ follows from the fact that dirty jobs are undesirable. If not, all workers would work in the job that offered greater utility. Therefore,

Proposition 1 *In a competitive labor market with clean and dirty jobs, equilibrium wages, w^{0*} and w^{1*}, are compensating for ξ^0 and ξ^1.*

Since w^{0*} and w^{1*} are compensating for ξ^0 and ξ^1, the willingness-to-pay for a clean job is:

$$WTP = w^{1*} - w^{0*} > 0.$$

Let w_i and ξ_i be individual level data generated by the model. Estimating a standard wage hedonic,

$$w_i = \alpha + \beta \xi_i + e_i \tag{3}$$

yields $\hat{\beta} = w^{1*} - w^{0*}$ and since w^{0*} and w^{1*} are compensating, $\hat{\beta} = WTP$.

In Rosen's (1986) more general treatment, workers have different utility functions and $w^{1*} - w^{0*}$ represents the willingness-to-pay for a clean job for workers who are indifferent between clean and dirty jobs. Other workers have a willingness-to-pay that is either strictly greater or strictly less and respectively work in clean or dirty jobs.

2.2 Oligopsony

Now consider the Hotelling labor market as described in Bhaskar *et al.* (2002). Equally able workers are uniformly distributed along the $[0,1]$ interval and two employers, 0 and 1, who pay wages w^0 and w^1 are located at either end. Workers pay a "transportation cost" of t per unit of distance to go to work so that a worker located at i pays ti to work for employer 0 and $t(1-i)$ to work for employer 1. Worker i gets utility that is the wage net of transportation costs or $U(w_i, \xi_i) = w_i - t\xi_i$ where $\xi_i = i$ when the worker works for employer 0 and $\xi_i = 1 - i$ when the worker works for employer 1. In a Hotelling labor market, a worker's WTP per unit of travel distance is is constant at t.

This notion of transportation cost can be interpreted literally as the actual cost of traveling to and from work. However, it can also be interpreted as a subjective measure of the extent to which a worker prefers one set of job characteristics over another set. Whether it involves physical distance or psychic distance, a worker may be willing to "travel" to the further, less preferred, employer for a sufficient wage premium.

Suppose that in equilibrium, job 0 pays w^{0*} and job 1 pays w^{1*}. Worker i must travel distance i to work at job 0 and $1-i$ to work at job 1. Let i^* be the worker that is indifferent between job 0 and job 1 so that $w^{1*} - t(1-i^*) = w^{0*} - ti^*$. In equilibrium,

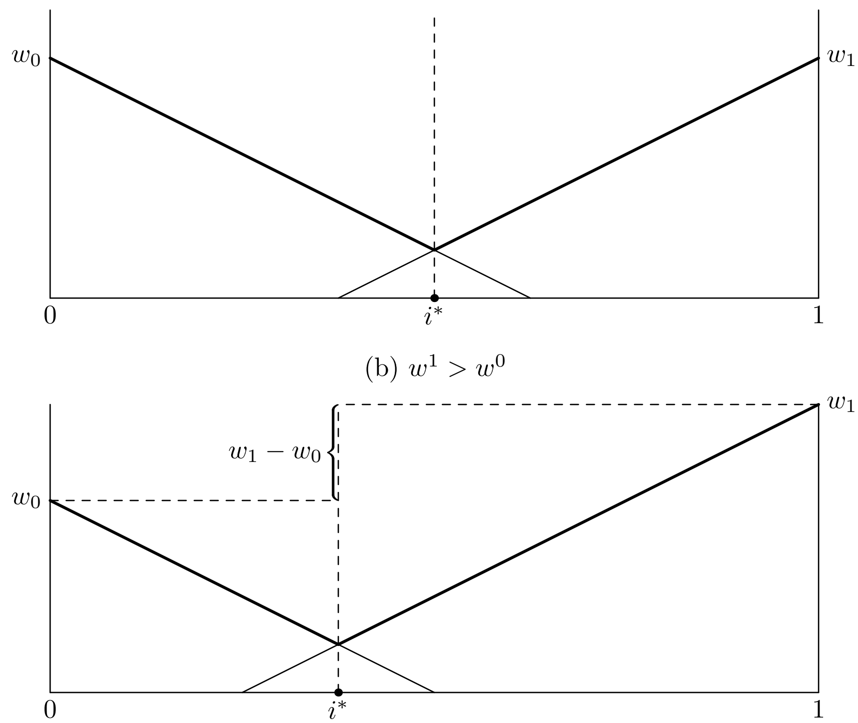
$$i^* = \begin{cases} 0 & \text{if } w^{1*} - w^{0*} > t \\ \frac{1}{2} - \frac{w^{1*} - w^{0*}}{2t} & \text{if } |w^{1*} - w^{0*}| \leq t \\ 1 & \text{if } w^{0*} - w^{1*} > t \end{cases}$$

where i^* workers work for employer 0 and $1 - i^*$ workers work for employer 1. Wage differentials can be compensating only if some workers work for each employer so assume that equilibrium wages are such that $|w^{1*} - w^{0*}| \leq t$. Since only worker i^* is indifferent between working for firm 0 and working for firm 1:

Figure 1: Hotelling job values

(a) $w^1 = w^0$

(b) $w^1 > w^0$



Proposition 2 *In a Hotelling labor market with linear transportation costs, equilibrium wages, w^{0*} and w^{1*}, are compensating only for workers located at i^*.*

In particular, for almost all workers, their wage differential is not compensating.

Figure 1 illustrates two examples where the downward and upward sloping line segments represent the worker's total utility. In each example, workers located at i^* are indifferent between working for employer 0 and working for employer 1; workers at every other location strictly prefer one or the other.

If employers are equally productive then they pay the same wage and evenly split the labor market (Figure 1a). Even though wages are identical, utilities, $U_i = w_i - t\xi_i$, are not. A standard compensating wage differential estimate (equation (3)) of the willingness-to-pay for commuting distance yields $\hat{\beta} = 0$, even though the true willingness-to-pay in the model is t.

Alternatively, when employers differ in productivity, the Hotelling model also generates biased hedonic estimates of the willingness-to-pay. For example, suppose employer 1 is more productive than employer 0 so that it offers a higher wage rate (i.e., $w^1 > w^0$) and employs a larger share of the labor market (Figure 1b). On average, workers employed at firm 0 travel shorter distances than those working for firm 1, so wages are positively correlated with commuting distance. As a result of this positive correlation, estimating a hedonic wage equation by OLS yields an estimated willingness-to-pay with the correct sign (i.e., $\hat{\beta} > 0$). However, this estimate is strictly bounded above by the true willingness-to-pay of t:

Proposition 3 *In a Hotelling labor market with linear transportation costs,* $E(\hat{\beta}) \ll WTP$.

Note that with the Hotelling labor market, there is a result analogous to Rosen's model where workers differ in their preferences over clean and dirty jobs. In particular, the wage differential, $w^{1*} - w^{0*}$, measures the WTP for the marginal worker, i^* between working for employer 0 and employer 1. However, there is an important difference. In Rosen's clean/dirty job model the wage difference captures the marginal worker's WTP for a clean job. With the Hotelling model, since employers are ex ante identical with regards to their non-wage amenities, the WTP for worker i^* only reveals that the employer offering the higher wage is more productive.[1]

Finally, this simple Hotelling analysis can be easily extended to n employers uniformly distributed about a unit circle (Bhaskar and To, 2003). If employers differ in their productivity, then wages are similarly dispersed and a wage differential is compensating only if there is a worker that is indifferent between them.

2.3 Unemployed search with amenities

Consider a labor analog of the Diamond (1971) model where employers are heterogeneous and like in the earlier Rosen (1986) model, are either clean or dirty. Time is discrete and workers employed with a dirty employer suffer

[1]If the econometrician knows i^* and the structure of worker preferences then $w^{1*} - w^{0*}$ can be used by the econometrician to infer the value of t. But since travel costs are a metaphor for preference heterogeneity, it is unlikely that the econometrician can observe where in preference space worker i^* is located.

a disutility of d. Fraction $0 < \gamma < 1$ employers are clean and $1 - \gamma$ are dirty. Workers are identical and get utility b when unemployed and a worker employed at wage w gets utility $U(w, \xi) = w - d\xi$ where $\xi = 0$ for a clean job and $\xi = 1$ for a dirty job. Since utility is linear, willingness-to-pay is constant and:

$$WTP = d.$$

Unemployed workers receive a job offer with probability λ_u and employed workers are laid-off with probability δ.

Since all workers are identical, they have reservation utilities for clean and dirty jobs that offer identical net utilities, R^0 and $R^1 = R^0 + d$. Consider a candidate pair of wage offer distributions $F^0(w)$ and $F^1(w)$ for clean and dirty jobs. Clean and dirty employers will deviate from $F^0(w)$ and $F^1(w)$ by offering clean wage $w^0 = R^0$ and dirty wage $w^1 = R^1$: wages lower than R^0 and R^1 will be rejected and they do not need to offer wages higher than R^0 and R^1 to have their offer accepted. But if all firms offer R^0 and R^1 then reservation utilities fall — if every employer offers net utility R^0, workers are willing to accept slightly lower wages because delay in matching is costly. As a result, employers reduce their common offers to these new reservation utilities. This continues until every employer offers wages $w^{0*} = b$ and $w^{1*} = b + d$.

Like the competitive Rosen model,

Proposition 4 *In a Diamond (1971) labor market with a disamenity,* $w^{1*} - w^{0*} = WTP$.

Finally, as shown by Hwang *et al.* (1998), in a Burdett and Mortensen (1998) style on-the-job search model with job amenities, job values are dispersed.[2] In particular, the equilibrium has the property that every employer of type j offers amenity level ξ_j but wage offers are drawn from a smooth distribution that has no gaps in its support. In other words, since amenity levels and wages are dispersed, wage differentials are not compensating. Moreover, hedonic estimates yield downward biased MWP estimates that can in some parameterizations have incorrect signs.

[2] Equilibrium job values, $v = w - d\xi$, are distributed according to the smooth distribution $G(v)$ where $G(v)$ has no gaps in its support.

3 Conclusions

In this paper we consider the question of when wage differentials are compensating. That is, when do observed wage differences compensate workers for differences in job amenities. In general, wage differences are compensating when there is job dispersion in equilibrium. For example in competitive labor markets (Rosen, 1986) or unemployed job search markets (Diamond, 1971) total job values are not dispersed and wage differences are compensating. But if labor markets are oligopsonistic (Bhaskar *et al.*, 2002; Bhaskar and To, 2003) or workers can search while on-the-job (Burdett and Mortensen, 1998; Hwang *et al.*, 1998) total job values are dispersed and wage differences are not generally compensating.

It is interesting to note that in this Diamond labor market, search frictions are maximal in the Ridder and van den Berg (2003) sense: they define $\kappa = \lambda_e/\lambda_l$ as a measure of the degree of search frictions — frictions are high when κ is low and frictions are low when κ is high. In the Diamond model, $\lambda_e = 0$ so that $\kappa = 0$ and search frictions are maximal. Nevertheless the Diamond monopsony outcome ensures that wage differences reveal workers' valuation of a clean job. That is, search frictions alone are not sufficient to bias OLS estimates of the marginal-willingness-to-pay.

A Appendix

Proof of Proposition 3: Given wages w^0 and w^1, i^* solves $w^0 - ti^* = w^1 - t(1-i^*)$ provided that $|w^1 - w^0| \leq t$. Assume that parameters are such that we have an interior equilibrium so that $w^1 - w^0 = t(1-2i^*)$.
Each $i \in [0,1]$ corresponds to a wage/disamenity pair. Given w^0, w^1 we know the location of the marginal worker, i^*. Since $i \sim U[0,1]$, w and ξ are random variables with distribution

$$w = \begin{cases} w^0 & \text{with probability } i^* \\ w^1 & \text{with probability } 1-i^* \end{cases}$$

and

$$\xi \sim \begin{cases} U[0,i^*] & \text{if } w = w^0 \\ U[0,1-i^*] & \text{if } w = w^1 \end{cases}.$$

For a linear, least-squares regression with a single independent variable, i.e., (3), it is well known that:

$$\hat{\beta} = \frac{\mathrm{cov}(w, \xi)}{\mathrm{var}(\xi)}.$$

Computing the covariance between w and ξ:

$$\begin{aligned}
\mathrm{cov}(w, \xi) &= E(w\xi) - E(w)E(\xi) \\
&= \left(i^* w^0 \int\limits_0^{i^*} \frac{\xi}{i^*} d\xi + (1 - i^*) w^1 \int\limits_0^{1-i^*} \frac{\xi}{1 - i^*} d\xi \right) - \\
&\quad (i^* w^0 + (1 - i^*) w^1) \left(i^* \int\limits_0^{i^*} \frac{\xi}{i^*} d\xi + (1 - i^*) \int\limits_0^{1-i^*} \frac{\xi}{1 - i^*} d\xi \right) \\
&= \left(w^0 \frac{i^{*2}}{2} + w^1 \frac{(1 - i^*)^2}{2} \right) - (i^* w^0 + (1 - i^*) w^1) \left(\frac{i^{*2}}{2} + \frac{(1 - i^*)^2}{2} \right) \\
&= \frac{i^*(1 - i^*)(2i^* - 1)}{2} (w^0 - w^1)
\end{aligned}$$

and since $w^1 - w^0 = t(1 - 2i^*)$,

$$\mathrm{cov}(w, \xi) = \frac{i^*(1 - i^*)(2i^* - 1)^2}{2} t.$$

Similarly, computing the variance of ξ,

$$\begin{aligned}
\mathrm{var}(\xi) &= E(\xi^2) - E(\xi)^2 \\
&= \left(i^* \int\limits_0^{i^*} \frac{\xi^2}{i^*} d\xi + (1 - i^*) \int\limits_0^{1-i^*} \frac{\xi^2}{1 - i^*} d\xi \right) - \left(i^* \int\limits_0^{i^*} \frac{\xi}{i^*} d\xi + (1 - i^*) \int\limits_0^{1-i^*} \frac{\xi}{1 - i^*} d\xi \right)^2 \\
&= \left(\frac{i^{*3}}{3} + \frac{(1 - i^*)^3}{3} \right) - \left(\frac{i^{*2}}{2} + \frac{(1 - i^*)^2}{2} \right)^2 \\
&= \frac{1}{12} - (1 - i^*)^2 i^{*2}.
\end{aligned}$$

As a result,

$$\hat{\beta} = \frac{6i^*(1 - i^*)(2i^* - 1)^2}{1 - 12(1 - i^*)^2 i^{*2}} t$$

This is non-negative for any $i^* \in [0, 1]$ and reaches its maximum value of $t/2$ when $i^* = (3 \pm \sqrt{3})/6$.

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